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Long-Term Competition for Product Awareness with Learning from Friends

Qiang Gong, Yujing Xu, and Huanxing Yang[‡]

Abstract

We study a dynamic model of price competition with differentiated products in which new generations of consumers acquire information about available products from their friends of previous generations. The social network, which links consumers across generations, affects the evolution of consumers' awareness of products and firms' long-term (steady-state) market shares. Focusing on steady-state equilibria, we examine how the structure of the social network - including connectivity and homophily - influences market shares, pricing, and welfare.

Key Words: Learning from friends; Social network; Price competition; Market Shares; Differentiated products

JEL Codes: D83; D85; L13; L14

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1 Introduction

In markets with differentiated products, consumers are often not fully aware of all available products. One important channel through which consumers learn about available products is by learning from friends who have previously purchased one of the products. This process of learning from friends, or “word of mouth,” greatly affects consumers’ purchasing behavior as shown by empirical evidence.¹ Given the importance of learning from friends, it is natural to study how the linking pattern among friends, or the structure of social networks, affects competition between firms and the resulting social welfare.

An important feature of the word-of-mouth learning is its dynamic nature. In particular, today’s market shares, through learning from friends, affect how well informed tomorrow’s consumers are, which in turn affects consumers’ purchasing decisions and market shares tomorrow. This dynamic learning process means that the evolution of consumers’ information status and purchasing behavior are jointly determined. As a result, when setting prices, profit-maximizing firms will internalize the effect of their current prices on demand in future periods. Moreover, this also implies that firms will pay extra attention to market shares, because current market shares not only affect firms’ profits today, but also affect future market shares and thus future profits through the dynamic learning channel. In the real world, firms do seem to emphasize current market shares relative to current profits. For instance, according to Farris et al. (2010), 67% of senior marketing managers and executives regard market share as an essential performance indicator in itself. Anecdotal evidence also suggests that many business leaders target market share (instead of profit) when setting business strategies (Edeling and Himme, 2018). Moreover, empirical evidence also indicates that market shares are more important for firms’ long run profits in industries where learning from friends plays an important role.²

This article studies how the structure of social networks affects firms’ competition and social welfare in the presence of the above dynamic learning process. In particular, there is a sequence of generations of consumers, and the linking pattern between each adjacent generations is described by a social network. The purchasing behavior of an old generation, through learning from friends, affects the information status and thus the purchasing behavior of the next generation, and so forth. The structure of the social network is important in that it influences the joint evolution of consumers’ information status and purchasing behavior. By incorporating this dynamic learning process, our article makes the following contributions.

¹For empirical evidence regarding the effect of “word of mouth,” see Keaveney (1995) on banking, Chevalier and Mayzlin (2006) on book sales, Chintagunta et al. (2010) on entertainment, and Luca (2016) on restaurant choices.

²As documented in Edeling and Himme (2018), the market share–financial performance elasticity is higher in markets where learning from friends plays an important role, such as B2C firms (relative to B2B firms), manufacturing firms (relative to service firms), and emerging markets (relative to the US markets).

First, our article provides an alternative rationale as to why firms seemingly overemphasize on market shares: with word-of-mouth learning among consumers, current market shares affect firms' future demand.³ In doing so, we also shed light on how this emphasis on market shares varies across industries/products, depending on the importance of word-of-mouth learning and the relevant social network structure. Second, regarding the impacts of social network structures on firms' competition and social welfare, our article qualitatively overturns the predictions that are drawn from the earlier static analysis (Campbell, 2019), which will be elaborated later.

Our model has two long-lived firms located at the opposite ends on a Hotelling line and competing in prices. We allow the two firms to be asymmetric. Each generation of consumers is uniformly distributed on the Hotelling line and lives for one period only. Each consumer has a unit demand, and a necessary condition for a consumer to buy a product is that he is aware of that product. For each generation, a fixed proportion of consumers is exogenously fully informed (aware of both firms' products). For the remaining (endogenous) consumers, they learn about the existence of products from their friends of the previous generation: an endogenous consumer becomes aware of a product if he has a friend of the previous generation who purchased that product.⁴ As a result, some endogenous consumers are partially informed (aware of one product only). The number of friends a consumer has is governed by the structure of the social network. In the basic model, we focus on the case of *random connections*, in which the location of each friend is uniformly drawn at random. We then study the case of *homophily*, under which friends are more likely to have similar locations.

The structure of the network determines how the purchasing behavior of one generation translates into the distribution of information status of the next generation, which in turn determines the purchasing behavior of that generation. We focus on steady-state (long-term) market shares. Firms set prices at the very beginning, which remain fixed in all later periods. Firms' prices induce steady-state market shares through learning from friends, and we assume that firms' objectives are to maximize their steady-state (long-term) profits.⁵

We show that the structure of the network affects firms' pricing decisions in a non-trivial way through the dynamic social learning process. In our model, the firms compete not only for the fully informed consumers in the current period, but also for the partially informed consumers in future periods. By setting a lower price and expanding its full-information market share, through dynamic social learning, a firm gains more partially informed consumers who are aware of its product only in future periods. Interpreting more broadly, in our model

³The existing literature (see the references in Bhattacharya et al., 2022) identifies three main channels through which higher market shares might positively affect firms' profitability: market power, operating efficiency, and quality signaling. Our article suggests an alternative channel of word-of-mouth learning.

⁴We assume that each consumer has at least one friend and thus is aware of at least one product.

⁵Both assumptions, perfectly patient firms and fixed prices, will be relaxed in later extensions.

current market share serves as a sort of “advertising” through the word-of-mouth learning: a bigger market share today means more consumers will be aware of a firm’s product in the future, which implies an even bigger market share in the future. Due to this channel, the intensity of competition crucially depends on how sensitive the steady-state demand is to the full-information market share, which in turn depends on the structure of the social network.

Our formal analysis mainly focuses on the case with symmetric firms. In the basic model with random connections, we derived the following main results. First, under general networks competition is more intense compared to the Hotelling benchmark. Second, the intensity of competition is non-monotonic in network connectivity (the number of friends consumers have). In particular, under the least connected network (the single-friend network under which each consumer has exactly one friend) and the most connected network (the infinite-friend network under which each consumer has an infinite number of friends), the equilibrium price is the highest and coincides with that in the Hotelling benchmark. Under any other generic network, the equilibrium price is lower, and the equilibrium price is non-monotonic in network connectivity.⁶ Finally, consumer surplus is also non-monotonic in network connectivity. These results may have important implications for empirical research, especially for industries (or products) in which word-of-mouth learning is important, as using the sensitivity of current-period demand to price would underestimate the sensitivity of the (long-run) demand that firms actually care about.

The above results are mainly driven by the dynamic learning process. Due to its presence, firms also compete for partially informed consumers in the future by affecting the full-information market share. This leads to more intense competition in our model than in standard Hotelling model where firms compete only for fully informed consumers in one period. The dynamic learning process is also responsible for our non-monotonicity result. In particular, an increase in network connectivity induces two effects. First, as the network becomes more connected or each consumer has more friends, more consumers become fully informed and there are fewer partially informed consumers. Consequently, the competition for the partially informed consumers in future periods is softened. We refer to this as the *volume effect* of network connectivity. On the other hand, network connectivity also has the following *ratio effect*. As the network becomes more connected, for the same increase in a firm’s full-information market share (from the equal split in equilibrium), among the partially informed consumers the ratio of the consumers who are informed of the product of the firm in question only to those who are informed of the other firm’s product only increases. This ratio effect tends to make the steady-state demand more sensitive to the full-information market share and intensifies competition. These two effects work in opposite directions, giving rise to the non-monotonicity of

⁶Among the *k-friend* networks (each consumer has exactly k friends), the price is the lowest under the two-friend network and increases in k for $k \geq 2$.

the intensity of competition in network connectivity.

The non-monotonicity of consumer surplus in network connectivity is mainly due to the non-monotonicity of price. Intuitively, as the network becomes more connected, each consumer gets “more” information, and thus more consumers are fully informed and fewer consumers buy “wrong” (less preferred) products. This information effect improves total welfare and consumer surplus. However, network connectivity also affects the equilibrium price. Because the equilibrium price is non-monotonic in network connectivity, consumer surplus is also non-monotonic. In particular, under relatively well-connected networks, the equilibrium price increases in connectivity, and this pricing effect dominates the information effect so that overall consumer surplus is decreasing in network connectivity.

We also find that an increase in the proportion of exogenously fully informed consumers leads to a higher equilibrium price. The underlying reason is that, with a larger proportion of exogenously fully informed consumers, there are fewer partially informed consumers in the future to compete for, which softens competition. This is a surprising result as it is the opposite to the prediction in standard models (e.g., Varian, 1980), where more fully informed consumers intensifies competition and lowers prices. The reason behind different predictions is again that in standard models firms compete for fully informed consumers only, whereas in our dynamic model firms also compete for partially informed consumers due to dynamic learning.

In the model with homophily, our main finding is that the equilibrium price is monotonically increasing in the degree of homophily; that is, homophily softens competition. As a result, consumer surplus could be decreasing in the degree of homophily. Again, the main driving force is the dynamic learning process. Intuitively, homophily leads to the following information effect: with homophily consumers are more likely to be aware of the “right” (preferred) products, as their friends are more likely to have similar horizontal tastes and thus have bought the “right” products; in other words, the “quality” of information received from friends improves. This means that, even if a firm cuts price and expands its full-information market share, it will induce fewer partially informed consumers to “wrongly” buy its product. Therefore, a higher degree of homophily softens the competition for partially informed consumers and raises price.

When firms are asymmetric, we find that the dynamic learning process amplifies the advantage of the advantaged firm (say firm 1). Specifically, compared to the Hotelling benchmark, in equilibrium firm 1 has a larger market share and firm 2 has a smaller market share and a lower price. The underlying reason is that with a full-information market share bigger than $1/2$, firm 1 can gain additional partially informed consumers through the dynamic learning process, which also forces firm 2 to reduce its price. We also find that homophily dampens the advantage of the advantaged firm: firm 1’s equilibrium market share is decreasing in the degree of homophily. The underlying reason is again that homophily dampens the dynamic

learning effect among partially informed consumers.

Finally, we also study two extensions. In the first extension, firms have a general discount factor and they maximize discounted profits. In the second extension, we allow firms to change prices across periods in a simple two-period model. In both settings, our main result, the non-monotonicity of prices in network connectivity, is largely robust. The assumption of fixed prices across periods will be further discussed in Section 7.

Related Literature The closest article to ours is Campbell (2019), who studies how the social network structure affects firms' competition and social welfare in a Hotelling model with consumers learning from friends about the available products. The main difference is that his model is essentially static; there are only two generations, the purchasing behavior of the old generation is exogenously given, and firms compete only for the new generation of consumers once. In contrast, our model is dynamic, with the joint evolution of consumers' information status and purchasing behavior across generations endogenously determined. Another difference is that Campbell (2019) focuses solely on symmetric firms, whereas our article also considers asymmetric firms.

As argued in the Introduction, a dynamic model is a better fit for real-world markets. Moreover, by making the model dynamic, our article generates predictions that are qualitatively different from those in Campbell (2019). Specifically, with random connections, in his model competition is less intense compared to the Hotelling benchmark, and a more connected network always intensifies competition and improves consumer welfare. With homophily, his model predicts that the degree of homophily does not affect the equilibrium price and homophily always improves consumer welfare. These results are qualitatively different from the predictions of our model. The differences in predictions are mainly driven by the dynamic learning channel in our model. With the dynamic learning process, firms also compete for partially informed consumers in the future by affecting the full-information market share, a feature absent in Campbell's (2019) static analysis. In our extension with a general discount factor δ , Campbell (2019) can be incorporated as a special case with $\delta = 0$. Finally, incorporating the dynamic learning effect is also important for empirical researchers and competition authorities, the ignoring of which could lead to misleading estimations.⁷

More broadly, our article is related to several strands of literature in industrial organization that study settings in which some consumers are not fully informed about available products or prices. One strand of literature studies consumers' search for product information (e.g., Varian, 1980; Stahl, 1989). Another strand considers firms' advertising strategies in informing

⁷For instance, ignoring the dynamic learning effect will underestimate the intensity of competition between firms; by attributing lower observed prices to a lower degree of horizontal differentiation, it also leads to underestimation of the degree of horizontal differentiation.

consumers about their products (Butters, 1977; see Bagwell, 2007, for an excellent survey). In terms of modeling, Grossman and Shapiro (1984) is particularly related. In their model with differentiated products, firms need to send costly advertisements in order to inform consumers about their products. Their focus is on firms' advertising strategy and its impact on pricing and welfare.

There is a large literature on learning through word of mouth. For instance, Smallwood and Conlisk (1979) consider a model in which new consumers, when selecting a new product, sample the products used by the existing consumers in the population and mimic their choices; and they study how firms' current market shares affect long-run adoption among consumers. Ellison and Fudenberg (1995) and Banerjee and Fudenberg (2004) study how word-of-mouth learning affects agents' choice between alternatives with stochastic payoffs in non-market environments.⁸ Overall, in this literature how the structure of the social network affects the market outcomes received little attention.

Our article is also related to a growing literature on industrial organization which studies firm behavior when either information is diffused through a social network or there is consumption externality between neighbors (see Bloch, 2016, for a survey). Some articles consider monopoly pricing (Bloch and Querou, 2013; Campbell, 2013; Fainmesser and Galeotti, 2016), some consider monopoly advertising/seeding (Galeotti and Goyal, 2009; Campbell et al., 2017), and some study oligopoly advertising/seeding (Bimpikis et al., 2016; Goyal et al., 2019). For articles studying oligopoly pricing, Aoyagi (2018), Chen et al. (2018), and Fainmesser and Galeotti (2020) consider models in which there are direct consumption externalities between neighboring consumers and firms are able to price discriminate based on consumers' network positions. Different from those models, in our model there is no consumption externality and we focus on the diffusion of product information via a social network. Galeotti (2010) develops a duopoly model in which firms produce a homogeneous good and consumers can get informed about prices by two channels: costly search and learning from friends. He also finds that the impact of network connectivity on the expected price is non-monotonic. However, the non-monotonicity result in his model is different from ours both in terms of directions and underlying mechanisms (see Section 3.2 for details). Relatedly, Kovac and Schmidt (2014) characterize market share dynamics in a Bertrand model with homogeneous products in which consumers learn about firms' prices from friends and firms play mixed pricing strategies. Different from these two models in which products are homogeneous and consumers learn prices from their friends, in our setting firms produce differentiated products and consumers learn about the existence of available products from their friends.

More recently, Campbell et al. (2020) developed a model in which consumers learn about

⁸Other dynamic pricing models with word-of-mouth learning include Rob and Fishman (2005) and Bergemann and Valimaki (2006).

the quality of experience goods from their friends. Their model and ours share some similar features. For instance, consumers are modelled as overlapping generations with the old generation serving as the source of information, and both articles focus on steady state. The main difference lies in different focus. Specifically, their article focuses on learning about qualities and does not consider price competition. In contrast, our article focuses on learning about the existence of available products and its impact on price competition.

Finally, in terms of the role played by market shares, our article is also related to the dynamic pricing literature with network externalities (e.g., Farrell and Saloner, 1986; Mitchell and Skrzypacz, 2006; Chen et al., 2009; Cabral, 2011). The general insight of this literature is that, relative to static competition, firms set lower prices in dynamic competition in order to expand their installed consumer bases. This is similar to our result that competition is generically more intense in the dynamic model than in the static Hotelling model. The main difference is that in this literature the network externality results from direct consumption externalities, whereas in our model the “network externality” (a bigger market share today leads to an even bigger market share in the future) is indirect and informational, which is generated through consumers’ learning from friends across generations. This difference leads to different predictions. For instance, although in that literature stronger consumption externalities lead to more intense competition, in our setting the intensity of competition is non-monotonic in network connectivity.

The rest of the article is organized as follows. Section 2 sets up the model. The basic model with symmetric firms and random connections is analyzed in Section 3. Section 4 studies the case with homophily. In Section 5 we consider asymmetric firms; and Section 6 studies two extensions: one with impatient firms and the other with prices changing across periods. Section 7 contains concluding remarks. All the proofs are relegated into the Appendix or the Online Appendix.⁹

2 Model

Time is discrete and denoted as T , and the horizon is infinite. There are two infinitely lived firms, firm 1 and 2, competing with each other in a Hotelling model; they are located at the two end points (firm 1 at location 0). In each period, there is a new generation of consumers active in the market, and they exit the market after one period. We index consumers by generation T . Each generation of consumers is of measure 1, and they are uniformly distributed on $[0, 1]$, with a consumer’s location indexed by $x \in [0, 1]$. Each consumer has a unit demand. A consumer at location x gets utility $V + \Delta - tx$ from buying firm 1’s product, and $V - t(1 - x)$ from

⁹The Online Appendix is available at <https://sites.google.com/site/huanxingyang1999/home/publications>.

buying firm 2's product. Here t is the per-unit transportation cost, and $\Delta \geq 0$ represents firm 1's advantage over firm 2. The marginal cost of each firm is normalized to 0. We assume V is large enough so that each consumer will buy exactly one product, and the question is which one. Finally, we assume that $\Delta < t$, which ensures that both firms are active in the market.

Next we introduce consumers' information status. For each generation T , a $\lambda \in (0, 1)$ proportion of consumers, independent of location x , are aware of both products through an exogenous process (for instance, through consumer search or firms' advertising campaigns).¹⁰ The remaining $1 - \lambda$ proportion of consumers, which we call *endogenous consumers*, are initially unaware of either products. They learn about the existence of a product from their friends of the previous generation $T - 1$ (old consumers who already purchased). The pattern of the social connections or friendship network is described by a distribution $\{p_k\}$, where p_k is a consumer's probability of having k friends of the old generation. We assume that $p_0 = 0$, which means that each consumer has at least one friend, thus is informed of at least one product. By having a friend of the old generation, a new consumer becomes informed of the product *purchased* by that friend in the last period (but not the other product), even if that friend is aware of both products. This assumption is reasonable for two reasons. First, observing a friend's action is easier than transmitting all the knowledge that the friend has. Second, in case that the friend is aware of both products, given that he purchased a particular product he must favor that product. This means that in communication he is more likely to mention or advocate the product he purchased only. Given this assumption, if a new consumer has two friends of the old generation who purchased different products, then the new consumer becomes aware of both products (i.e., fully informed). If all friends of a new consumer bought the same product, say product 1, then the new consumer is aware of product 1 only.¹¹

In the basic model, we will focus on the case of *random connections*. That is, the friends of a new consumer are uniformly drawn at random in terms of locations, independent of the new consumer's own location. We then study *homophily* in an extension, in which case a consumer and her friends are more likely to have similar locations. Denote $\psi_T(x)$ as the probability that a consumer of generation T at location x buys from firm 1, and ψ_T as the proportion of consumers of generation T who buy product 1. That is, firm 1's market share is ψ_T and that of firm 2 is $1 - \psi_T$ in period T .

Firms know the structure of the network, but does not observe the locations of individual consumers. In the very beginning (say period 0), the two firms set prices P_1 and P_2 simultane-

¹⁰ A positive fraction of exogenously fully informed consumers is needed to ensure that the steady-state market shares are non-degenerate (both firms have positive market shares). See footnote 12 for details.

¹¹ The feature of overlapping generations is only a convenient modeling device. More generally, in each period T a fraction of new consumers arrive and could consult their friends who already bought the products in earlier periods (all old consumers, not necessarily the ones who arrived in period $T - 1$). This general setting will be discussed in Section 7.

ously, which remain fixed in all later periods (this assumption will be relaxed in Section 6.2 and discussed further in Section 7). Given the friendship network across generations, consumers' information status about the products and the market share ψ_T , in general, will evolve across periods through learning from friends. We will focus on the *steady-state* (or long-term) market share, which satisfies $\psi_{T+1} = \psi_T \equiv \psi$. In short, given P_1 and P_2 , through learning from friends the market share will eventually reach a steady state. We assume that each firm's objective is to maximize its steady-state profit. One justification for this assumption is that firms are very patient.¹² In Section 6, we consider the case of impatient firms with a general discount factor δ .

Our model is closely related to Campbell (2019), with two main differences. First, Campbell (2019) considers symmetric firms ($\Delta = 0$) only, whereas we also consider asymmetric firms. Second and more importantly, in Campbell (2019) firms compete with each other in one period only, and the information status of the old-generation consumers is exogenously given. In contrast, in our model firms compete over time and the information status of each generation is endogenously derived. In terms of the technical analysis of the steady state, our model is also related to Campbell et al. (2019), which study how the structure of the social media network affects the prevalence of different types of media content. They also focus on steady state, under which the frequency that each type of message is forwarded remains unchanged over time. However, the research questions that our article addresses are very different from their article's.

To facilitate later comparisons, here we compute the equilibrium outcome in the standard Hotelling model ($\lambda = 1$ in our setting) as a benchmark. Using superscript H to denote the Hotelling outcomes, the equilibrium prices are $P_1^H = t + \frac{\Delta}{3}$ and $P_2^H = t - \frac{\Delta}{3}$, and the equilibrium market share of firm 1 is $\hat{x}^H = \frac{1}{2} + \frac{\Delta}{6t}$.

3 Random Connections

In this section we study the case of random connections. We first characterize steady-state equilibrium, and then examine the impact of network connectivity on equilibrium outcome.

Steady-state equilibrium

First observe that for each generation, a consumer's information status can be one of the following: fully informed, aware of product 1 only, or aware of product 2 only. With random

¹²Specifically, firm 1's average per-period profit (firm 2's is similar) is $\pi_1(P_1, P_2) = (1 - \delta) \sum_{T=1}^{\infty} \delta^{T-1} P_1 \psi_T$, where δ is the discount factor. When $\delta \rightarrow 1$, $\pi_1(P_1, P_2)$ converges to $P_1 \psi$, the steady-state per-period profit. Otherwise, this assumption is still reasonable if, given any P_1 and P_2 , the steady state is reached rather quickly.

connections, a consumer's information status is independent of her location x . Our first step is to derive the steady state market share as a function of P_1 and P_2 . For this purpose, we need to trace the distribution of consumers' information status across periods. Let $\phi_{F,T}$ be the proportion of consumers of generation T who are fully informed, and $\phi_{i,T}$, $i = 1, 2$, be the proportion of consumers of generation T who are aware of product i only.

Define \hat{x} as the cutoff consumer (in terms of location) who is fully informed and indifferent between product 1 and 2. Because consumers live for one period only, this \hat{x} coincides with the cutoff in the Hotelling model:

$$\hat{x} = \frac{1}{2} + \frac{P_2 - P_1 + \Delta}{2t}. \quad (1)$$

Because firm 1 has an advantage with $\Delta \geq 0$, in equilibrium, $\hat{x} \in [1/2, 1)$. Note that \hat{x} is also firm 1's market share if all consumers were fully informed. Given ψ_T , \hat{x} , and λ , we have the following transition equations:

$$\begin{aligned} \phi_{1,T+1} &= (1 - \lambda) \sum_k p_k \psi_T^k, & \phi_{2,T+1} &= (1 - \lambda) \sum_k p_k (1 - \psi_T)^k, \\ \phi_{F,T+1} &= 1 - \phi_{1,T+1} - \phi_{2,T+1} = \lambda + (1 - \lambda) \left[1 - \sum_k p_k [\psi_T^k + (1 - \psi_T)^k] \right], \\ \psi_{T+1} &= \phi_{1,T+1} + \hat{x} \phi_{F,T+1}. \end{aligned}$$

In the first (second) equation, the consumers in generation $T + 1$ who are informed of product 1 (2) only must be someone: (i) who are not exogenously fully informed, and (ii) whose friends of generation T all bought product 1 (2) only. In the third equation, the fully informed consumers in generation $T + 1$ include consumers who are exogenously fully informed and endogenous consumers whose friends of generation T bought different products. In the last equation, the fraction of consumers buying product 1 in generation $T + 1$ consists of two groups: consumers who are aware of product 1 only, and the fully informed consumers who prefer product 1.

Combining the above equations, we have the following transition equation, which specifies ψ_{T+1} as a function of ψ_T :

$$\psi_{T+1} = \hat{x} + (1 - \lambda) \sum_k p_k [(1 - \hat{x})(\psi_T)^k - \hat{x}(1 - \psi_T)^k] \equiv H(\psi_T). \quad (2)$$

The three terms of $H(\psi_T)$ in (2) can be interpreted as follows. The first term \hat{x} is firm 1's full-information market share. The second term is the fraction of consumers who "wrongly"

bought product 1 (they prefer firm 2’s product but are informed of product 1 only); firm 1 gains this portion of market share relative to the full-information benchmark. Similarly, the third term is the fraction of consumers who “wrongly” bought product 2; and firm 1 loses this portion of market share relative to the full-information benchmark. Taken together the second and third terms, it represents firm 1’s net gain of market share among the partially informed consumers relative to the full-information benchmark.

In the steady state, $\psi_{T+1} = \psi_T \equiv \psi$; that is, $\psi = H(\psi)$. By (2), the steady-state equation can be explicitly written as

$$\psi = \hat{x} + (1 - \lambda) \sum_k p_k [(1 - \hat{x})\psi^k - \hat{x}(1 - \psi)^k] \equiv H(\psi). \quad (3)$$

Note that P_1 and P_2 affect the steady-state market share ψ only through their effects on \hat{x} . The next lemma shows that \hat{x} induces a unique steady-state market share ψ .

Lemma 1 *Given a full-information market share (for firm 1) $\hat{x} \in [1/2, 1)$, there is a unique steady-state market share ψ , which is globally stable and satisfies $\psi \in [\hat{x}, 1)$. Moreover, ψ is strictly increasing in \hat{x} .*

The result that $\psi \geq \hat{x}$ is intuitive. To see this, suppose firm 1’s market share equals $\hat{x} \geq 1/2$. Then, due to learning from friends, there will be (weakly) more consumers who wrongly purchase product 1 than those who wrongly purchase product 2, as firm 1 has a larger full-information market share. This indicates that firm 1’s steady-state market share ψ would be (weakly) bigger than \hat{x} .

The relationship between ψ and \hat{x} depends on the network structure $\{p_k\}$. To make the relationship more transparent, we will pay special attention to regular networks under which $\{p_k\}$ is degenerate. In particular, we define a *k-friend network* as a regular network under which every consumer has exactly k friends ($p_k = 1$ for some $k \geq 1$). Among *k-friend networks* there are two extreme networks. The first one is the *single-friend network*, under which each consumer has exactly one friend ($p_1 = 1$). The second one is the *infinite-friend network*, under which each consumer has infinitely many friends ($k \rightarrow \infty$). We use the term generic networks for networks other than the single-friend network and the infinite-friend network.¹³ The next lemma sheds some light on the relationship between ψ and \hat{x} under different networks.

Lemma 2 *(i) Under the single-friend network or the infinite-friend network, the steady-state market share ψ equals to \hat{x} , the full-information market share. (ii) Under generic networks,*

¹³Under any generic network, if $\lambda = 0$, then by (3) it can be verified that $\psi = 1$ whenever $\hat{x} \in (\frac{1}{2}, 1)$.

$\psi > \hat{x}$ if $\hat{x} > 1/2$. (iii) Under k -friend networks, fixing $\hat{x} \in (1/2, 1)$, ψ strictly decreases in k if $k \geq 2$.

Lemma 2 holds the key in understanding later results. The result regarding the infinite-friend network is quite intuitive: under this network, all consumers in each generation are fully informed, and therefore the steady-state market share ψ equals to the full-information one, \hat{x} . Under the single-friend network, when $\psi = \hat{x}$, the fraction of endogenous consumers who wrongly buy product 1 exactly equals to those who wrongly buy product 2, both of which are $(1 - \hat{x})\hat{x}$. Thus firm 1's steady-state market share is \hat{x} .¹⁴ For k -friend networks with $k \geq 2$, as k increases (the network becomes more connected), the total number of partially informed consumers $((1 - \lambda)[\psi^k + (1 - \psi)^k])$ decreases, but the ratio of the number of consumers informed of product 1 only to those informed of product 2 only, $(\frac{\psi}{1 - \psi})^k$, increases because $\psi > \hat{x} > 1/2$. The first effect tends to decrease firm 1's steady-state market share, and the second effect works in the opposite direction. It turns out that when $k \geq 2$, the first effect dominates, and firm 1's net gain of market share among the partially informed consumers decreases in k . Therefore, among k -friend networks, the two-friend network leads to the biggest steady-state market share for firm 1.

Now we are ready to characterize steady-state equilibria, and we will focus on pure strategy equilibria. Firm 1's and firm 2's profits in steady state are $\pi_1(P_1, P_2) = P_1\psi(\hat{x})$ and $\pi_2(P_1, P_2) = P_2[1 - \psi(\hat{x})]$, respectively. Each firm i chooses P_i , given P_j , to maximize its steady-state profit. The first-order conditions yield (with P_i^e being the equilibrium prices)

$$P_1^e = \frac{2t\psi}{\frac{d\psi}{d\hat{x}}} \text{ and } P_2^e = \frac{2t(1 - \psi)}{\frac{d\psi}{d\hat{x}}}, \quad (4)$$

where, by (3),

$$\frac{d\psi}{d\hat{x}} = \frac{1 - (1 - \lambda) \sum_k p_k [(\psi)^k + (1 - \psi)^k]}{1 - (1 - \lambda) \sum_k k p_k [(1 - \hat{x})(\psi)^{k-1} + \hat{x}(1 - \psi)^{k-1}]}. \quad (5)$$

Notice that $d\psi/d\hat{x} > 0$ (by Lemma 1) captures the sensitivity of ψ to \hat{x} and largely determines the intensity of competition. Combining (1), (4), and (5), we have the following pricing equation:

$$\hat{x} = \frac{1}{2} + \frac{\Delta}{2t} - (2\psi - 1) \frac{1 - (1 - \lambda) \sum_k k p_k [(1 - \hat{x})(\psi)^{k-1} + \hat{x}(1 - \psi)^{k-1}]}{1 - (1 - \lambda) \sum_k p_k [(\psi)^k + (1 - \psi)^k]}. \quad (6)$$

¹⁴Specifically, for k -friend networks,

$$H(\hat{x}) - \hat{x} = (1 - \lambda)(1 - \hat{x})\hat{x}[\hat{x}^{k-1} - (1 - \hat{x})^{k-1}].$$

Equations (3) and (6), with two unknowns (\hat{x} and ψ), jointly determine steady-state equilibria. Denote (\hat{x}_e, ψ_e) as an equilibrium pair of (\hat{x}, ψ) .¹⁵

Proposition 1 *There is a unique solution satisfying both the steady-state equation (3) and the pricing equation (6), with the equilibrium full-information market share $\hat{x}_e \in [\frac{1}{2}, \frac{1}{2} + \frac{\Delta}{2t}]$. If the two firms are symmetric ($\Delta = 0$), then both \hat{x}_e and the equilibrium steady-state market share ψ_e are symmetric: $\hat{x}_e = \psi_e = 1/2$. The solution is the unique steady-state equilibrium under either of the following conditions: (i) the network $\{p_k\}$ is the single-friend network or a well-connected network;¹⁶ (ii) the fraction of exogenously fully informed consumers λ is relatively large.*

Due to the coexistence of fully informed consumers and partially informed consumers, it is well-known that firms' profit functions may not be well-behaved. In particular, firms may charge high prices targeting partially informed consumers only. This means that the first-order conditions may not be sufficient, and may cause the non-existence of pure strategy equilibria. Conditions (i) and (ii) specified in Proposition 1 are sufficient conditions to avoid this problem: under either condition $\psi(\hat{x})$ is convex when $\hat{x} \leq 1/2$ and concave when $\hat{x} \geq 1/2$,¹⁷ so that the second-order conditions are satisfied globally. Intuitively, under either condition in the limit the model converges to the Hotelling benchmark. These two conditions are far from being necessary for the existence of equilibrium. Even when λ is relatively small so that the second-order conditions are not satisfied globally, each firm's profit function could still be single-peaked, meaning that the solution satisfying the first-order conditions is indeed the equilibrium.¹⁸

The impact of network connectivity

In this subsection we focus on *symmetric firms* with $\Delta = 0$. As Campbell (2019) focuses solely on symmetric firms, the comparison in this subsection will reveal clearly how introducing steady state (or long-term) demand affects results. With $\Delta = 0$, the equilibrium is symmetric with $\hat{x}_e = \psi_e = 1/2$, $P_1^e = P_2^e \equiv P^e = t / (\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2})$, and (5) becomes

$$\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2} = \frac{1 - (1 - \lambda) \sum_k p_k (\frac{1}{2})^{k-1}}{1 - (1 - \lambda) \sum_k k p_k (\frac{1}{2})^{k-1}}. \quad (7)$$

¹⁵From (\hat{x}_e, ψ_e) , we can recover the equilibrium prices P_1^e and P_2^e based on (4) and (5).

¹⁶A network is well-connected if each consumer has at least \bar{k} friends, with \bar{k} being large.

¹⁷See Lemma A1 in the Online Appendix.

¹⁸Consider the following examples. Network 1 has $p_1 = p_2 = p_3 = 1/3$ and network 2 has $p_2 = p_3 = p_4 = p_5 = 1/4$, with $\lambda = 1/4$, $t = 1$, and Δ being either 0 or $1/4$. For each case (four cases in total), firm i 's profit function $\pi_i(P_i, P_{j_e})$ is single-peaked at P_{i_e} , though $\pi_i(P_i, P_{j_e})$ is convex in P_i when P_i is large.

The expression of the equilibrium price reveals that the intensity of competition depends on the sensitivity of ψ to \hat{x} , evaluated at \hat{x}_e .

To examine how the network structure affects the equilibrium price, we define connectivity in terms of first-order stochastic dominance (FOSD): a network $\{p''_k\}$ is more connected than $\{p'_k\}$ if $\{p''_k\}$ FOSD $\{p'_k\}$.

Proposition 2 *The equilibrium price is not monotonic in network connectivity. In particular, the following results hold. (i) Under either the single-friend network or the infinite-friend network, the equilibrium price $P^e = t$, the transportation cost, but under any other generic network $P^e < t$. (ii) Among the k -friend networks, the equilibrium price P^e is the lowest under the 2-friend network: it decreases when k increases from 1 to 2 and strictly increases in k for $k \geq 2$.*

Part (i) of Proposition 2 shows that under generic networks competition is more intense than in the standard Hotelling model. This is because of the dynamic learning effect in our model: a bigger market share today (obtained by cutting price) will translate into a bigger steady-state market share in the future through the dynamic learning channel. As mentioned earlier, this is reminiscent of the result in the dynamic pricing literature with network externalities. The difference is that in our model the “network externality” is indirectly generated through consumers’ learning from friends across generations.

The non-monotonicity result in part (ii) can be understood in light of part (iii) of Lemma 2. In more general terms, we can decompose the net effect of network connectivity on $d\psi/d\hat{x}$ into two effects: the *volume effect* and the *ratio effect*. Specifically, as the network becomes more connected, more consumers are fully informed and the fraction of partially informed consumers decreases. This volume effect tends to reduce the sensitivity of ψ to \hat{x} and soften competition, as there are fewer partially informed consumers to compete for. On the other hand, a more connected network means that, for the same (non-equilibrium) market share $\psi > 1/2$, the ratio of the number of consumers informed of product 1 only to those informed of product 2 only will increase.¹⁹ This means that firm 1 can get a bigger market share among the partially informed consumers through dynamic social learning. This ratio effect tends to increase the sensitivity of ψ to \hat{x} and intensify competition. Because the volume effect and the ratio effect work in opposite directions, the equilibrium price is not monotonic in network connectivity. Among the k -friend networks, it turns out that the ratio effect dominates when k increases from 1 to 2, and the volume effect dominates when $k \geq 2$.

Proposition 2 is qualitatively different from the corresponding result in Campbell (2019), where the equilibrium price monotonically decreases as the friendship network becomes more

¹⁹ As mentioned earlier, under k -friend networks this ratio is $(\frac{\psi}{1-\psi})^k$, which is increasing in k when $\psi > 1/2$.

connected. The main reason for the difference is that in Campbell’s (2019) one-period model, firms compete for fully informed consumers only. As the network becomes more connected, because the fraction of fully informed consumers increases, competition intensifies and the price decreases. In contrast, in our model firms are competing not only for fully informed consumers in the current period, but also for partially informed consumers in future periods. A bigger market share of firm i today (achieved by setting a lower price), through the friendship network, will lead to more (fewer) partially informed consumers who are informed of product i (j) only in the next period, which translates to an even bigger market share in the future. This dynamic channel is absent in Campbell (2019).

In a duopoly model with homogeneous goods, Galeotti (2010) also shows that the average equilibrium price is non-monotonic in network connectivity. As mentioned earlier, in his model consumers can get fully informed through costly search and learning from friends. As consumers have more friends, its direct effect is to increase the fraction of fully informed consumers, given any fixed level of search effort. However, it also crowds out consumers’ search effort as the free-riding problem becomes more severe, which tends to reduce the fraction of fully informed consumers. These two opposite effects lead to the non-monotonicity of price in network connectivity.²⁰ The non-monotonicity result in our article is due to a different mechanism: an increase in connectivity leads to the volume effect and the ratio effect about the partially informed consumers, and the combination of these two opposite effects generates non-monotonicity. Another qualitative difference is that in Galeotti (2010) the relationship between price and network connectivity exhibits an inverse U shape (the price reaches the maximum for some intermediate level of connectivity), whereas in our model the relationship is U shape (the price reaches the minimum for an intermediate level of connectivity). This is because in his (static) model firms compete for fully informed consumers only, whereas in our model firms also compete for partially informed consumers due to the dynamic learning channel.

To capture more precisely the dynamic learning effect, in the symmetric equilibrium we compute the sensitivity of firm 1’s current period demand to its price, $|\frac{\partial Q_1}{\partial P_1}|$, and the sensitivity of firm 1’s steady-state (long-run) demand to its price, $|\frac{\partial \psi}{\partial P_1}|$. In particular,

$$|\frac{\partial Q_1}{\partial P_1}| = \frac{1 - (1 - \lambda) \sum_k p_k (\frac{1}{2})^{k-1}}{2t},$$

where the numerator is precisely the fraction of fully informed consumers, as in the current

²⁰In a related article on monopoly advertising/seeding, Galeotti and Goyal (2009) show that the relationship between the level of advertising and network connectivity is also non-monotonic. The reason is that advertising and word of mouth (WOM) can be substitutes or complements, depending on the cost of advertising.

period only those consumers are sensitive to prices. Using (7), we have

$$\left| \frac{\partial \psi}{\partial P_1} \right| = \frac{\left| \frac{\partial Q_1}{\partial P_1} \right|}{1 - (1 - \lambda) \sum_k k p_k \left(\frac{1}{2}\right)^{k-1}} > \left| \frac{\partial Q_1}{\partial P_1} \right|.$$

Observe that the long-run demand is more sensitive to price than the current-period demand is, and the difference between the two sensitivities exactly captures the dynamic learning effect. This also suggests that using the sensitivity of current-period demand to price in empirical studies to fit firms' behaviors could be misleading. In the real world, what firms try to maximize is more likely to be long-run profits rather than short-run profits, and thus the sensitivity of long-run demand to price is a more appropriate measure. In other words, using the sensitivity of current-period demand would underestimate the sensitivity of the (long-run) demand that firms actually care about. Our model also provides a justification for firms' seemingly puzzling overemphasis on current market shares over the current profits (see the empirical evidence mentioned earlier in the Introduction). This is because, due to the dynamic learning effect, a firm's current market share also affects the evolution of its future market share and thus its long-run profits.²¹

Proposition 3 *Suppose networks are generic. (i) If network $\{p''_k\}$ FOSD network $\{p'_k\}$ and the probabilities of having a single friend is the same under the two networks ($p''_1 = p'_1$), then the equilibrium price is higher under network $\{p''_k\}$ than under network $\{p'_k\}$. (ii) The equilibrium price is increasing in λ , the fraction of exogenously fully informed consumers.*

For generic (non-regular) networks, the relationship between network connectivity and equilibrium price follows a pattern similar to the one under regular (the k -friend) networks. Part (i) of Proposition 3 shows that, if a FOSD change in connectivity does not reduce p_1 (thus puts more probabilities on higher number of links), then it softens competition and increases the equilibrium price. However, if a FOSD change reduces p_1 , then it may intensify competition and reduce prices (see Example 1). For the same reason, the equilibrium price is not monotonic with respect to changes of mean-preserving spread (see Example 2).²²

Part (ii) of Proposition 3 is a surprising result. In standard models of price competition (such as the search models of Varian (1980) and Stahl (1989)), an increase in the fraction of exogenously fully informed consumers typically intensifies competition and reduces prices

²¹Our explanation is different from Bendle and Vandenbosch (2014), which explains why competitor orientation can persist and even thrive based on evolutionary games. Our explanation is also consistent with the empirical finding mentioned in footnote 2.

²²In Campbell (2019), a mean-preserving spread in $\{p_k\}$ always softens competition and increases prices.

(Anderson and Renault (2000) is an exception).²³ By contrast, in our model the result is the opposite. The underlying reason, as mentioned earlier, is that firms in our model also compete for partially informed consumers of future generations through the dynamic social learning. When λ increases, there are fewer partially informed consumers to compete for, or the steady-state demand becomes less sensitive to prices. As a result, competition is softened and prices increase.

Example 1 (*FOSD*). Let $\lambda = 1/4$, $\{p'_k\} = \{p'_1 = p'_2 = 1/2\}$, and $\{p''_k\} = \{p''_1 = p''_2 = p''_3 = 1/3\}$. Note that $\{p''_k\}$ FOSD $\{p'_k\}$. The values of $\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2}$ under $\{p'_k\}$ and $\{p''_k\}$ are $7/4$ and $9/5$, respectively, leading to a lower equilibrium price under $\{p''_k\}$.

Example 2 (*Mean-preserving spread*). Let $\lambda = 1/4$, $\{p_k\}_A = \{p_2 = 1\}$, and $\{p_k\}_B = \{p_1 = p_2 = p_3 = 1/3\}$, $\{p_k\}_C = \{p_4 = 1\}$, and $\{p_k\}_D = \{p_3 = p_5 = 1/2\}$. Note that $\{p_k\}_B$ is a mean-preserving spread of $\{p_k\}_A$, and $\{p_k\}_D$ is also a mean-preserving spread of $\{p_k\}_C$. The values of $\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2}$ under $\{p_k\}_A$ and $\{p_k\}_B$ are $5/2$ and $9/5$, respectively; thus the equilibrium price is higher under $\{p_k\}_B$ than under $\{p_k\}_A$. The values of $\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2}$ under $\{p_k\}_C$ and $\{p_k\}_D$ are 1.45 and 1.47 , respectively; thus the equilibrium price is lower under $\{p_k\}_D$ than under $\{p_k\}_C$.

Next we consider the impact of network connectivity on social welfare. Denote the total welfare and consumer surplus in the steady-state equilibrium as W and CS , respectively. In particular,

$$\begin{aligned} W &= V - 2\left\{\frac{t}{4}\left[\frac{1}{2} - (1-\lambda)\sum_k p_k\left(\frac{1}{2}\right)^{k+1}\right] + \frac{3}{4}t(1-\lambda)\sum_k p_k\left(\frac{1}{2}\right)^{k+1}\right\} \\ &= V - \frac{t}{4} - t(1-\lambda)\sum_k p_k\left(\frac{1}{2}\right)^{k+1}. \end{aligned} \tag{8}$$

Total welfare W equals to V minus the total transportation costs incurred. The latter includes the transportation costs incurred by the consumers who bought their right products, which is captured by the first term in the braces in the first line; among these consumers the average transportation costs per consumer is $t/4$. The second term in the braces is the total transportation costs incurred by the consumers who bought wrong products; among these consumers, the average transportation costs per consumer is $3t/4$. As to consumer surplus, $CS = W - P^e$,

²³ Anderson and Renault (2000) found a similar result that equilibrium price increases in the fraction of informed consumers, but in a different setup with a different definition of being informed. Their consumers search for both prices and match values; informed consumers know their match values before they search prices.

which by (8) can be written as

$$CS = V - \frac{t}{4} - t \frac{1 - (1 - \lambda) \sum_k k p_k (\frac{1}{2})^{k-1}}{1 - (1 - \lambda) \sum_k p_k (\frac{1}{2})^{k-1}} - t(1 - \lambda) \sum_k p_k (\frac{1}{2})^{k+1}. \quad (9)$$

Proposition 4 *Total welfare W is increasing in network connectivity. However, consumer surplus CS is not monotonic as the network connectivity increases. In particular, among the k -friend networks, (i) CS is higher under the two-friend network than under the single-friend network; (ii) when $k \geq 3$, CS is decreasing in k ; (iii) when k increases from 2 to 3, CS decreases if the fraction of exogenously fully informed consumers $\lambda \leq \sqrt{113} - 10 \simeq 0.63$, and increases otherwise.*

The result that total welfare always increases in network connectivity is easy to understand. As the network becomes more connected, more consumers are fully informed and thus fewer consumers buy wrong products, which reduces the total transportation costs and increases total welfare. As to consumer surplus, besides the *information effect* mentioned above, there is also a *pricing effect* as the equilibrium price changes with network connectivity. As the pricing effect is not monotonic, the overall effect of network connectivity on consumer surplus is not monotonic either. Among the k -friend networks, as k changes from 1 to 2, the price decreases; thus the pricing effect and the information effect work in the same direction and CS increases. With $k \geq 2$, the price is increasing in k , which means that the pricing effect and the information effect work in opposite directions. It turns out that when $k \geq 3$ the pricing effect always dominates so that CS is decreasing in k . Intuitively, when the network is already relatively well connected, the fraction of consumers who buy wrong products is already small. Thus a further increase in connectivity only leads to a small decrease in the number of consumers buying wrong products, and hence the information effect is small. On the other hand, a price increase induced by an increase in connectivity hurts all consumers. Therefore, the pricing effect dominates when k is relatively large.

Comparing our predictions and those of Campbell (2019), although the result regarding total welfare is the same, the predictions on consumer surplus are quite different, as the pricing effects across two models are qualitatively different. In particular, Campbell (2019) predicts that consumer surplus always increases in connectivity, whereas our model predicts that it increases in connectivity when the network is sparsely connected, but decreases in connectivity otherwise.

[Figure 1 is about here]

To see the magnitude of the effect of network connectivity on the equilibrium price, social welfare and consumer surplus, consider the following examples. We define networks $\{p_k\}$ with $p_{k-1} = p_k = p_{k+1} = 1/3$ as networks with three-point distributions, which is indexed by k . With $t = 1$, $\lambda = 1/4$, and $V = 2$, Figure 1 illustrates how the equilibrium price and welfare change with k among the k -friend networks and networks with three-point distributions. For the k -friend networks, the price reduction and the increase in CS between the single-friend network and the two-friend network are significant. When k is between 2 and 6, as k increases the price increases considerably and CS decreases considerably. When k is bigger than 7, both the price increase and CS reduction become relatively insignificant as k increases. This example shows that, among not very well connected regular networks, an increase in connectivity could have quantitatively significant impacts on both price and welfare. A similar pattern holds under the networks with three-point distributions, though the changes in price and CS become smaller (relative to the regular networks) as k increases. Overall, the two examples indicate that the dynamic learning effect is quantitatively important under not very well connected networks.

4 Homophily

A prevalent feature of social networks is homophily. That is, individuals tend to have friends who are similar to themselves.²⁴ In our context, homophily is reflected in the pattern that consumers at similar locations in the product space are more likely to be friends. We use parameter $\alpha \in [0, 1]$ to capture the degree of homophily. In particular, for each consumer at location x , with probability $1 - \alpha$ a friend is drawn uniformly at random from location $[0, 1]$, and with probability α a friend is drawn from location $[x - \varepsilon, x + \varepsilon]$, with $\varepsilon \geq 0$ but very small. With this setup, a bigger α implies a higher degree of homophily. The purpose of this section is to study the impact of homophily on pricing and welfare.

Steady-state equilibrium

With homophily, consumers become more likely to be aware of their preferred products, as their friends are more likely to have bought their preferred products. In order to trace the evolution of consumers' information status and purchasing behavior, we need to separate consumers into two types (groups). For a given \hat{x} , denote L type consumers as those with $x \leq \hat{x}$ (who buy product 1 if fully informed), and R type consumers as those with $x > \hat{x}$ (who buy product 2 if fully informed). Type L consumers and type R consumers differ in information received in

²⁴See McPherson, Smith-Lovin and Cook (2001).

the ex ante sense, as their friends' purchasing behaviors are different statistically. In order to make the analysis tractable, we further assume that $\varepsilon \rightarrow 0$. With $\varepsilon \rightarrow 0$, the same process governs all L type consumers' information status and purchasing behaviors,²⁵ and the same holds among all R type consumers.

To proceed, we first introduce notations. Denote $\psi_{L,T}$ ($\psi_{R,T}$) as the the proportions of L (R) type consumers of generation T who buy product 1, $\psi_T = \hat{x}\psi_{L,T} + (1 - \hat{x})\psi_{R,T}$ as the market share of firm 1 in period T (which is also the probability that a random consumer of generation T buys product 1), and $\phi_{jh,T}$ as the proportion of type h ($h = L, R$) consumers of generation T who are informed of product j only. The transition equations (across generations) are listed below.

$$\begin{aligned}\phi_{1R,T+1} &= (1 - \lambda) \sum_k p_k [(1 - \alpha)\psi_T + \alpha\psi_{R,T}]^k, \\ \phi_{2L,T+1} &= (1 - \lambda) \sum_k p_k [1 - (1 - \alpha)\psi_T - \alpha\psi_{L,T}]^k, \\ \psi_{L,T+1} &= 1 - \phi_{2L,T+1}, \quad \psi_{R,T+1} = \phi_{1R,T+1}.\end{aligned}$$

In the first equation, $(1 - \alpha)\psi_T + \alpha\psi_{R,T}$ is the probability that a type R consumer's (of generation $T + 1$) friend (of generation T) buys product 1, and this consumer is informed of product 1 only if all his generation T friends bought product 1. Similarly, in the second equation, $1 - (1 - \alpha)\psi_T - \alpha\psi_{L,T}$ is the probability that a type L consumer's (of generation $T + 1$) friend (of generation T) buys product 2, and this consumer is informed of product 2 only if all his generation T friends bought product 2. The above consumers in generation $T + 1$ are informed of "wrong" products only, and thus will buy "wrong" products, as indicated by the third and fourth equations.

Let ψ_L , ψ_R , and ψ be the steady-state market shares of firm 1. The steady-state conditions require $\psi_{L,T+1} = \psi_{L,T} \equiv \psi_L$ and $\psi_{R,T+1} = \psi_{R,T} \equiv \psi_R$. Using the above equations, we have the following steady-state equations:

$$1 - (1 - \lambda) \sum_k p_k [1 - (1 - \alpha)\psi - \alpha\psi_L]^k = \psi_L, \quad (10)$$

$$(1 - \lambda) \sum_k p_k [(1 - \alpha)\psi + \alpha\psi_R]^k = \psi_R, \quad (11)$$

$$\hat{x}\psi_L + (1 - \hat{x})\psi_R = \psi. \quad (12)$$

²⁵If $\varepsilon > 0$ and does not converge to 0, then the information received (ex ante) among the same type consumers depends on their locations. A type L located very close to \hat{x} would receive information that is statistically different from a type L located close to 0, as the former's neighbors include some type R consumers. This would make the analysis intractable.

The above system consists of three equations with three unknowns, ψ , ψ_L and ψ_R . We can solve these three endogenous variables as a function of \hat{x} . Equations (10), (11), and (12) can also be combined as

$$\psi = \hat{x} + (1 - \lambda) \sum_k p_k \{(1 - \hat{x})[(1 - \alpha)\psi + \alpha\psi_R]^k - \hat{x}[1 - (1 - \alpha)\psi - \alpha\psi_L]^k\}. \quad (13)$$

As indicated by (13), ψ equals the full-information market share \hat{x} plus the fraction of consumers “wrongly” bought product 1 minus the fraction of consumers “wrongly” bought product 2. When $\alpha = 0$ (no homophily), (13) boils down to (3) in Section 3. The next three lemmas present useful properties regarding the steady-state market shares for a fixed \hat{x} .

Lemma 3 *Fix the steady-state market share $\psi \in [1/2, 1)$. The following results hold for ψ_L (ψ_R), the fraction of type L (R) consumers who buy firm 1’s product. (i) There is a unique ψ_L satisfying (10) and a unique ψ_R satisfying (11), with $0 < \psi_R < \psi < \psi_L < 1$. (ii) Both ψ_L and ψ_R are increasing in ψ ; ψ_L is increasing and ψ_R is decreasing in the degree of homophily α . (iii) $\psi_R \geq 1 - \psi_L$.*

Observe that ψ_R is the probability that a type R consumer “wrongly” buy product 1, and $1 - \psi_L$ is the probability that a type L consumer “wrongly” buy product 2. As the degree of homophily α increases, the probability that each type of consumers are informed of wrong products only decreases, and thus both ψ_R and $1 - \psi_L$ decrease (as shown in part (ii) of Lemma 3). The reason for $\psi_R \geq 1 - \psi_L$ (as in part (iii)) is that the market share of firm 1, ψ , is weakly larger than $1/2$. Due to the component of random connection, it means that the probability that a type L consumer is aware of product 1 is weakly higher than the probability that a type R consumer is aware of product 2. As a result, compared to a type L consumer, a type R consumer is more likely to buy the wrong product.

Lemma 4 *Fix the full-information market share $\hat{x} \in [1/2, 1)$. (i) There is a unique steady-state market share ψ (and ψ_L and ψ_R as well), which satisfies $\psi \in [\hat{x}, 1)$. (ii) $\psi_R < \psi < \psi_L$ and ψ strictly increases in \hat{x} .*

By Lemma 4, a given full-information market share \hat{x} induces a unique steady-state market share ψ . The next lemma shows how network connectivity and the degree of homophily affect ψ .

Lemma 5 *Fix the full-information market share $\hat{x} \in [1/2, 1)$. (i) Under the single-friend network or the infinite-friend network, or any other generic network but with the degree of homophily $\alpha = 1$, the steady-state market share $\psi = \hat{x}$. (ii) Suppose $\hat{x} \in (1/2, 1)$. Under any generic network with $\alpha < 1$, $\psi > \hat{x}$ and ψ decreases in α .*

The most revealing result in Lemma 5 is that, under generic networks, firm 1's steady-state market share ψ decreases in the degree of homophily α . To understand the intuition, let us consider two polar cases: random connection and extreme homophily ($\alpha = 1$). Under random connection, each consumer is more likely to be aware of product 1 than product 2 due to the nature of random connection and the fact that $\hat{x} > 1/2$. Then there are more R type consumers wrongly buying product 1 than L type consumers wrongly buying product 2, resulting in ψ being bigger than \hat{x} . However, in the case of extreme homophily, there is no consumers buying wrong products, because every friend of a L (R) type consumer is of L (R) type, and thus every consumer is aware of the right product in steady state. As a result, the steady-state market share $\psi = \hat{x}$. The general case ($\alpha \in (0, 1)$) is just a combination of the above two polar cases. As α increases, each type of consumers become more likely to be aware of their preferred products, and thus there are fewer consumers buying wrong products. This means that \hat{x} ($> 1/2$) translates into a smaller additional market share for firm 1 among partially informed consumers, i.e., ψ decreases.

[Figure 2 is about here]

Figure 2 illustrates the relationship between α and the shape of the $\psi(\hat{x})$ curve. In the figure, $\lambda = 1/4$, $p_1 = p_2 = p_3 = 1/3$ in network 1 and $p_2 = p_3 = p_4 = p_5 = 1/4$ in network 2. Under both networks, the $\psi(\hat{x})$ curve becomes more straight as α increases.

Next we endogenize \hat{x} and characterize steady-state equilibria. The pricing equations (first-order conditions) have the same form as in the case of random connections:

$$\begin{aligned} P_1 &= \frac{2t\psi}{\frac{d\psi}{d\hat{x}}}, P_2 = \frac{2t(1-\psi)}{\frac{d\psi}{d\hat{x}}}, \\ \hat{x} &= \frac{1}{2} + \frac{\Delta}{2t} - \frac{2\psi - 1}{\frac{d\psi}{d\hat{x}}}, \end{aligned} \quad (14)$$

where by (10), (11) and (12),

$$\begin{aligned} \frac{d\psi}{d\hat{x}} &= \frac{\psi_L - \psi_R}{1 - [\hat{x} \frac{\partial \psi_L}{\partial \psi} + (1 - \hat{x}) \frac{\partial \psi_R}{\partial \psi}]} \\ &= \frac{1 - (1 - \lambda)[\sum_k p_k (z_L^k + z_R^k)]}{1 - (1 - \lambda)(1 - \alpha) [\hat{x} \frac{\sum_k k p_k z_L^{k-1}}{1 - (1 - \lambda)\alpha \sum_k k p_k z_L^{k-1}} + (1 - \hat{x}) \frac{\sum_k k p_k z_R^{k-1}}{1 - (1 - \lambda)\alpha \sum_k k p_k z_R^{k-1}}]}, \end{aligned} \quad (15)$$

where $z_L \equiv 1 - (1 - \alpha)\psi - \alpha\psi_L$ and $z_R \equiv (1 - \alpha)\psi + \alpha\psi_R$. By Lemma 4, $\frac{d\psi}{d\hat{x}}$ is strictly positive.

Equations (14) and (10)-(12) jointly determine the equilibrium pair of (\hat{x}, ψ) , which we denote as (\hat{x}_e, ψ_e) .

Proposition 5 *There is a unique solution satisfying both the steady-state equations (10)-(12) and the pricing equation (14), with the equilibrium full-information market share $\hat{x}_e \in [\frac{1}{2}, \frac{1}{2} + \frac{\Delta}{2t}]$. If the two firms are symmetric ($\Delta = 0$), then both \hat{x}_e and the equilibrium steady-state market share ψ_e are symmetric: $\hat{x}_e = \psi_e = 1/2$. The solution is the unique steady-state equilibrium under either of the following conditions: (i) the network $\{p_k\}$ is the single-friend network or a well-connected network; (ii) either the fraction of exogenously fully informed consumers λ is relatively large, or the degree of homophily α is relatively high.*

Similar to Proposition 1, conditions (i) and (ii) specified in Proposition 5 are sufficient conditions under which $\psi(\hat{x})$ is convex when $\hat{x} \leq 1/2$ and concave when $\hat{x} \geq 1/2$,²⁶ so that the second-order conditions are satisfied globally. Intuitively, under either condition in the limit the model converges to the Hotelling benchmark. Again, these two conditions are far from being necessary. Even when λ is relatively small and α is relatively low so that the second-order conditions are not satisfied globally, each firm's profit function could still be single-peaked, meaning that the solution satisfying the first-order conditions is indeed the equilibrium.²⁷

The impact of homophily

In this subsection we focus on *symmetric firms* ($\Delta = 0$). Proposition 5 shows that the unique equilibrium is symmetric, with $\psi_e = \hat{x}_e = 1/2$, and $P_1^e = P_2^e \equiv P^e = t / (\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2})$. Moreover, by (10) and (11), $\psi_L = 1 - \psi_R$, and $z_L = z_R$. Then (15) can be simplified as:

$$\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2} = \frac{(1 - 2\psi_R)\{1 - (1 - \lambda)\alpha \sum_k k p_k [(1 - \alpha)/2 + \alpha\psi_R]^{k-1}\}}{1 - (1 - \lambda) \sum_k k p_k [(1 - \alpha)/2 + \alpha\psi_R]^{k-1}}. \quad (16)$$

The next proposition characterizes the impact of homophily on the equilibrium price.

Proposition 6 *(i) If network $\{p_k\}$ is the single-friend network or the infinite-friend network, then for any degree of homophily α , the equilibrium price $P^e = t$, the transportation cost. (ii) Under any other generic network, $P_e < t$ (unless $\alpha = 1$) and is monotonically increasing in α .*

Proposition 6 shows that homophily softens competition and increases the equilibrium price.²⁸ This result differs significantly from the one in Campbell (2019), where the degree of

²⁶See Lemma A2 in the Online Appendix.

²⁷With $\Delta = 0$, in each of the four examples in Figure 2, each firm's profit function is single-peaked in its own price when the other firm's price is fixed at its candidate equilibrium price.

²⁸The impact of homophily is not restricted to market competition. See Galeotti and Mattozi (2011) on politicians' choice of policy platforms, Golub and Jackson (2012) on the speed of social learning and Campbell

homophily does not affect the equilibrium price. The underlying reason for the difference is again the dynamic learning effect, which is absent in Campbell’s (2019) model. The intuition for our result is already indicated in Lemma 5. As the degree of homophily α increases, each type of consumers (L or R) is more likely to be informed of their preferred products, and overall there are fewer consumers buying wrong products. This means that, if a firm cuts its price and thus expands its full-information market share \hat{x} above $1/2$, it can only induce fewer partially informed consumers to wrongly buy its product. As a result, the dynamic learning effect is dampened, and the steady-state market share becomes closer and less sensitive to the full-information market share \hat{x} , which softens competition and increases the price.

Next we examine the impact of homophily on welfare. Similar to earlier analysis, total welfare W equals the gains from trade minus the total transportation costs incurred. Specifically,

$$W = V - 2\left[\frac{t}{4}\psi_L + \frac{3t}{4}\frac{1}{2}\psi_R\right] = V - \frac{t}{4} - \frac{t}{2}\psi_R. \quad (17)$$

The second equality follows from the fact that $\psi_L = 1 - \psi_R$. In the expression of (17), $t/4$ is the total transportation costs if all consumers buy right products, ψ_R is the fraction of consumers who buy “wrong” products, and on average each of these consumers suffers an additional transportation costs of $t/2$. Similarly, consumer surplus CS can be written as

$$CS = W - P^e = V - \frac{t}{4} - \frac{t}{2}\psi_R - t/\left(\frac{d\psi}{d\hat{x}}\Big|_{\hat{x}=1/2}\right). \quad (18)$$

Proposition 7 (i) Total welfare W is increasing in the degree of homophily α . (ii) Under the single-friend network or the infinite-friend network, consumer surplus CS is increasing in α . (iii) Under any other generic network $\{p_k\}$, if the probability of having a single friend p_1 is less than that of having exactly two friends p_2 , then CS is decreasing in α for $\alpha \leq 1/2$; if $p_1 = 0$, then CS is decreasing in α for $\alpha \leq 3/4$.

The result that homophily improves total welfare is intuitive. As the degree of homophily increases, consumers are more likely to be informed of their preferred products and thus fewer consumers buy wrong products, which decreases the total transportation costs and increases total welfare. As to consumer surplus, besides the information effect mentioned above, which always benefits consumers, there is a pricing effect which works in the opposite direction. In particular, homophily increases the price and thus hurts consumers. Part (iii) of Proposition 7 shows that the overall effect of homophily on consumer welfare is negative if α is not too large. The pricing effect tends to dominate, because an increase in α only prevents an additional

et al. (2019) on political polarization in the social media network.

fraction of consumers from buying wrong products, but the resulting increase in price hurts all consumers. Notice that this result is qualitatively different from Campbell (2019), in which homophily always improves consumer welfare. This is because the pricing effect of homophily is absent in his model.

[Figure 3 is about here]

In fact, numerical examples suggest that the overall effect of homophily on consumer welfare could be negative for the entire domain of α . Using the same networks as before with $t = 1$ and $\lambda = 1/4$, Figure 3 plots how the equilibrium price, total welfare, and consumer surplus change as α varies. Under both networks, consumer surplus is monotonically decreasing in α for any α .

5 Asymmetric Firms

In this section we consider asymmetric firms; that is, $0 < \Delta \leq t$. Due to asymmetry, in equilibrium firms will charge different prices and have different market shares. We first consider the case with random connections, and then consider the case with homophily.

Random connections

With random connections, we are mainly interested in how network connectivity affects firms' asymmetry in equilibrium.

Proposition 8 *(i) The single-friend network and the infinite-friend network lead to the same equilibrium prices and market shares, which coincide with those in the standard Hotelling model. (ii) Under generic networks, compared to the Hotelling benchmark, competition is more intense and the equilibrium market share of firm 1 (2) is higher (lower), and firm 2's equilibrium price is lower. (iii) Under the k -friend networks with $k \geq 2$ and firm 1's advantage Δ not being too large, firm 1's equilibrium market share ψ_e decreases in k .*

Part (ii) of Proposition 8 shows that, relative to the Hotelling benchmark, with learning from friends generically firm 1's advantage and firm 2's disadvantage are amplified. The underlying reason is again the dynamic learning across generations. With asymmetric firms, the full-information market share of firm 1 is strictly bigger than 1/2. Through learning from friends, firm 1 can gain additional market share among partially informed consumers. To

counter the reduced market share, firm 2 reduces its price. The result also indicates that the market shares and prices are not monotonic in network connectivity: the two extreme networks with the lowest and highest connectivity have the same market shares and prices, whereas for all other networks the market share ψ_e is higher and firm 2's price is lower. Part (iii) implies that, among the k -friend networks with $k \geq 2$ and when two firms are not too asymmetric, an increase in connectivity dampens the advantage of firm 1. The reason behind this result is part (iii) of Lemma 2. In particular, when $k \geq 2$, a further increase in k reduces firm 1's market share ψ for any given \hat{x} . This effect tends to reduce ψ_e as k increases.

The Online Appendix provides concrete examples under the k -friend networks, with the qualitative relationship between prices and k the same as in the case with symmetric firms. Broadly speaking, the general pattern is that if the initial network is already relatively well connected, then a further increase in connectivity would decrease firm 1's market share and soften competition, as the dynamic learning effect is dampened.

Homophily

With homophily, we focus on how the degree of homophily affects the asymmetry between firms in equilibrium.

Proposition 9 *Consider generic networks, and suppose that firm 1's advantage Δ is not too large. The equilibrium market share of firm 1, ψ_e , is decreasing in the degree of homophily α . Moreover, if $\alpha < 1$, then, relative to the Hotelling benchmark, in equilibrium ψ_e is higher and firm 2's price is lower.*

Proposition 9 shows that homophily dampens the advantage of firm 1 and the disadvantage of firm 2. The underlying intuition for this result is the same as that of part (ii) of Lemma 5: homophily dampens the dynamic learning effect. That is, as α increases, there are fewer consumers buying wrong products, which means that the same $\hat{x} > 1/2$ would translate into a smaller additional market share among partially informed consumers for firm 1. The Online Appendix provides concrete examples when Δ is large, in which the result in Proposition 9 holds globally. Overall, the conclusion is that an increase in the degree of homophily α softens competition and dampens firm 1's advantage.

6 Robustness

In the baseline model we have assumed that firms' objectives are maximizing steady-state profits and their prices, once set, are fixed in all periods; the former effectively assumes that

$\delta = 1$, where δ is firms' discount factor. In this section we will relax these two assumptions, and examine how our main results will change. For simplicity, we restrict attention to the case of symmetric firms with random connections.

Impatient firms

In this subsection we consider a general $\delta \in [0, 1]$, and each firm's objective is to maximize its discounted sum of profits by setting its price. To set the model rolling, we need to introduce generation 0 consumers (old generation in period 1). As firms are symmetric and we focus on symmetric equilibrium, we assume that 1/2 of generation 0 consumers bought from firm 1 (consistent with the steady-state market share). Then under any symmetric price the market share ψ_T will converge to the steady-state market share in period 1, which is 1/2. All the other assumptions are the same as in the baseline model. Note that our baseline model corresponds to the case that $\delta = 1$ (firms care only about long-run steady profits), and the model of Campbell (2019) corresponds to the case that $\delta = 0$ (firms care only about the profits in period 1).

Denote the symmetric equilibrium price as P^* . Sometimes we write the market shares explicitly as $\psi_T(P_1, P_2)$ and $\hat{x}(P_1, P_2)$, and denote $\psi_T^* = \psi_T(P^*, P^*)$ and $\hat{x}^* = \hat{x}(P^*, P^*)$. Note that on the equilibrium path, $\hat{x}^* = \psi_T^* = \frac{1}{2}$ for any T . Due to symmetry, we only need to consider firm 1's incentive. Specifically, firm 1 tries to maximize $\sum_{T=1}^{\infty} \delta^{T-1} P_1 \psi_T(P_1, P^*)$ by choosing P_1 , subject to the evolution of ψ_T (derived earlier in Section 3):

$$\psi_{T+1} = \hat{x} + (1 - \lambda) \sum_k p_k [(1 - \hat{x}) \psi_T^k - \hat{x} (1 - \psi_T)^k],$$

where more explicitly $\hat{x} = \hat{x}(P_1, P^*)$ and $\psi_T = \psi_T(P_1, P^*)$. In the symmetric equilibrium, the first-order condition for P_1 yields

$$\sum_{T=1}^{\infty} \delta^{T-1} \left[\frac{1}{2} + P^* \frac{\partial \psi_T^*}{\partial P_1} \right] = 0,$$

where $\frac{\partial \psi_T^*}{\partial P_1} = -\frac{1}{2t} \left[1 - (1 - \lambda) \sum_k p_k \left(\frac{1}{2}\right)^{k-1} \right] \sum_{j=1}^T \left((1 - \lambda) \sum_k p_k k \left(\frac{1}{2}\right)^{k-1} \right)^{j-1}$.

We can verify that the absolute value of $\frac{\partial \psi_T^*}{\partial P_1}$ strictly increases in T , and that

$$\lim_{T \rightarrow \infty} \frac{\partial \psi_T^*}{\partial P_1} = -\frac{1}{2t} \frac{1 - (1 - \lambda) \sum_k p_k \left(\frac{1}{2}\right)^{k-1}}{1 - (1 - \lambda) \sum_k p_k k \left(\frac{1}{2}\right)^{k-1}} = \frac{\partial \psi}{\partial P_1},$$

where ψ is the steady-state market share. From the above equations, we get

$$P^* = t \frac{1 - \delta(1 - \lambda) \sum_k p_k k (\frac{1}{2})^{k-1}}{1 - (1 - \lambda) \sum_k p_k (\frac{1}{2})^{k-1}}. \quad (19)$$

When $\delta = 1$, P^* in (19) coincides with the equilibrium price in our baseline model, and it coincides with the equilibrium price in Campbell (2019) when $\delta = 0$.

Proposition 10 *Consider symmetric firms with random connections and suppose the discount factor $\delta \in [0, 1]$. (i) The equilibrium price P^* decreases in firms' patience level δ . (ii) Among the k -friend networks, there exists a cutoff integer $\hat{k} \geq 2$ such that P^* decreases in k when $k \leq \hat{k}$ and increases in k when $k > \hat{k}$. The cutoff \hat{k} decreases in δ : it equals 2 when $\delta = 1$ and approaches ∞ when $\delta \rightarrow 0$.*

Part (i) of Proposition 10 shows that the model with a general δ nests Campbell (2019) and our baseline model as two extreme cases, with $\delta = 0$ and $\delta = 1$, respectively.²⁹ In particular, the equilibrium price continuously decreases as firms become more patient, and those in the two extreme cases are the lower and upper bounds. The underlying reason for this result is that the market shares (and hence profits) in later periods are more sensitive to prices ($\frac{\partial \psi_T^*}{\partial P_1}$ is increasing in T).³⁰ When firms become less patient, they put less weights on profits in later periods, which causes the equilibrium price to increase.

Part (ii) shows that, with a general δ , the non-monotonicity result regarding the effect of network connectivity on the equilibrium price (exhibits a U-shape) still holds as long as $\delta > 0$. Moreover, δ affects the size of the decreasing region (the relationship is negative) and that of the increasing region. When δ decreases, the decreasing region expands and the increasing region shrinks. When δ goes to 0, the increasing region disappears completely, which coincides with the result in Campbell (2019).

A general δ also affects Proposition 3. In particular, if δ is larger than some cutoff $\hat{\delta} \in (0, 1)$, then a FOSD change in network connectivity specified in part (i) still leads to an increase in the equilibrium price; but the result is reversed when $\delta < \hat{\delta}$. Similarly, in part (ii), the equilibrium price is increasing in λ (the fraction of exogenously fully informed consumers) if δ is large, and the result is reversed when δ is small. Regarding the welfare results in Proposition 4, with a general δ they are modified accordingly, as consumer surplus (CS) is mainly driven by the equilibrium price. Specifically, the relationship between network connectivity and CS still

²⁹By (19), under the single-friend network $P^* \geq t$, and under the infinite-friend network $P^* = t$.

³⁰If P_1 is a little bit lower than P^* , because the market share in period 0 is $1/2$, the market share ψ_T will increase monotonically until it reaches the steady-state level, which is higher than $1/2$.

exhibits an inverse U-shape if δ is not too small. Moreover, as δ decreases, the decreasing region gradually shrinks.³¹

Two-period model with changing prices

In this subsection, we allow firms to change prices across periods. For tractability, we focus on the case that there are only two periods. Again, we assume that 1/2 of generation 0 consumers bought from firm 1 ($\psi_0 = 1/2$). Each firm tries to maximize its discounted profits with discount factor δ . All the other assumptions are the same as in the baseline model. Again, we will focus on the symmetric equilibrium.

Denote $P_{i,T}$ as the price of firm i , \hat{x}_T as the full-information market share, and P_T^* as the symmetric equilibrium price, in period T , $T = 1, 2$. In particular,

$$\hat{x}_T = \frac{1}{2} + \frac{P_{2,T} - P_{1,T}}{2t}, \text{ and } \frac{d\hat{x}_T}{dP_{1,T}} = \frac{d(1 - \hat{x}_T)}{dP_{2,T}} = -\frac{1}{2t}.$$

Similar to the baseline model, the market share ψ_T evolves according to

$$\psi_T = \hat{x}_T \phi_{F,T} + (1 - \lambda) \sum_k p_k \psi_{T-1}^k, \text{ where } \phi_{F,T} = 1 - (1 - \lambda) \sum_k p_k [\psi_{T-1}^k + (1 - \psi_{T-1})^k]. \quad (20)$$

Here $\phi_{F,T}$ is the fraction of fully informed consumers, and $(1 - \lambda) \sum_k p_k \psi_{T-1}^k$ is the fraction of consumers informed of firm 1's product only, in period T . In the Appendix, we derive the equilibrium prices as follows:

$$P_2^* = \frac{t}{1 - (1 - \lambda) \sum_k p_k (\frac{1}{2})^{k-1}}; \quad P_1^* = t \frac{1 - \frac{2}{3}\delta(1 - \lambda) \sum_k p_k k (\frac{1}{2})^{k-1}}{1 - (1 - \lambda) \sum_k p_k (\frac{1}{2})^{k-1}}. \quad (21)$$

Proposition 11 *Consider the two-period model with symmetric firms, random connections, discount factor $\delta > 0$, and changing prices. (i) The symmetric equilibrium prices in period T , P_T^* ($T = 1, 2$), are given by (21), with $P_1^* < P_2^*$. (ii) Among the k -friend networks, the first-period price P_1^* is non-monotonic in k : it decreases in k when k is small and increases in k when k is large. The cutoff integer \tilde{k} is larger than \hat{k} , the cutoff in Proposition 10.*

The result that $P_1^* < P_2^*$ is intuitive. As period 2 is the last period, firms compete only for the fully informed consumers in that period.³² On the other hand, in the first period

³¹The proofs of the above results are available upon request.

³²Actually, P_2^* is the same as the equilibrium price in Campbell (2019).

firms compete not only for the fully informed consumers in period 1, but also for the partially informed consumers in period 2, as the market share in period 1 affects the fraction of partially informed consumers in period 2 through the dynamic learning channel. As a result, competition is more intense and the price is lower in period 1.

Part (ii) of Proposition 11 shows that the non-monotonicity result in our baseline model is robust when we allow prices to change across periods: P_1^* is non-monotonic in network connectivity. The underlying reason is the presence of the dynamic learning effect in period 1, as explained earlier. For the same δ , P_1^* is higher than P^* in the previous subsection because in the current setting there are only two periods, whereas in the previous setting the horizon is infinite. Based on the results in the two-period model, in a model with a longer horizon (say \bar{T} periods) and changing prices we conjecture that the following results will hold. First, the symmetric equilibrium prices increase over time (P_T^* increases in T), as in later periods the dynamic learning effect becomes weaker because there are fewer future periods left. Second, for any price P_T^* except for the last period \bar{T} , it is non-monotonic in network connectivity. This is because the dynamic learning effect is present in every period except for the last period \bar{T} .

7 Conclusion and Discussion

We study a dynamic model of price competition with differentiated products and word-of-mouth learning: each generation of consumers learns about available products from their friends of the previous generation. The social network, which links consumers across generations, affects the evolution of consumers' awareness of products and firms' long-term market shares. Focusing on steady-state equilibria, we examine how the structure of the social network influences market shares, prices, and welfare. Due to the dynamic learning process, firms' current market shares have long-run consequences on their future demand. As a result, firms also compete for partially informed consumers in future periods, and competition is more intense compared to the Hotelling benchmark. By incorporating this dynamic learning process, our article provides an alternative explanation as to why firms seemingly overemphasize on market shares, and sheds light on for which industries/products this overemphasis is more relevant.

In the basic model with random connections, we find that the intensity of competition is non-monotonic in network connectivity. In particular, under relatively well connected networks, a further increase in network connectivity softens competition. As a result, although total welfare is increasing in network connectivity, consumer surplus is non-monotonic because the impact of network connectivity on equilibrium price is not monotonic. In the model with homophily, we find that homophily softens competition, and that consumer welfare is non-monotonic in the degree of homophily. With asymmetric firms, the advantage of the advantaged

firm is amplified by the dynamic learning process, but it is dampened by homophily. As a robustness check, we also study two extensions: the first one considers the case of impatient firms, and in the second one we allow firms to change prices across periods in a two-period model. Our main results still hold qualitatively in both settings.

The assumption of fixed prices across periods makes our baseline model tractable. Although the two-period model with changing prices illustrates the robustness of our main results in the baseline model, we are not certain about whether our results will change in an infinite horizon model with flexible prices. It is highly desirable to fully work out such a model,³³ and we leave it for future research.³⁴ Another assumption of our model is that the friends of each new generation of consumers are of the previous generation only. At the expense of technical complications, we can extend the model to settings in which the friends of each new generation are from previous generations. For instance, generation T consumers could be friends with the $T - 1$ and $T - 2$ generations. However, extending our model to these settings will not qualitatively change our results, because firms are still competing for partially informed consumers in future periods due to the dynamic learning process. In these new settings, compared to our basic model, the steady state will be reached more slowly, because market shares in earlier periods will have more persistent impact on the demand in later periods. As a result, we conjecture that the steady-state demand will be more sensitive to the full-information market share and prices, which leads to more intense competition.

³³We conjecture that, in such a setting with symmetric firms, the price in symmetric Markov equilibrium will be stationary across periods.

³⁴Our model assumed that a consumer is only informed of the product purchased by her friend. Alternatively, a consumer could become informed of all the products that her friend is aware of. Consequently, consumers' information improves over time, and all consumers will be fully informed in the long-run, implying that the steady-state equilibrium will be the same as the standard Hotelling benchmark.

Appendix

Proof of Lemma 1.

Proof. We first show that, given \hat{x} , there is a steady-state ψ , at which the $H(\psi)$ curve crosses the 45-degree line. By (3), it is clear that $H(\psi)$ is continuous in ψ . Moreover,

$$\lim_{\psi \rightarrow 0} H(\psi) = \lambda \hat{x} > 0, \quad \lim_{\psi \rightarrow 1} H(\psi) = 1 - \lambda(1 - \hat{x}) < 1.$$

Therefore, there exists a $\psi \in (0, 1)$ satisfying $\psi = H(\psi)$.

For the uniqueness of ψ , it suffices to show that $\frac{\partial^3 H}{\partial \psi^3} \geq 0$ for all ψ within $(0, 1)$, which implies that $H(\psi)$ crosses the 45-degree line at most once within domain $(0, 1)$. By (3),

$$\frac{\partial^3 H}{\partial \psi^3} \propto \sum_{k \geq 3} p_k k(k-1)(k-2)[(1-\hat{x})\psi^{k-3} + \hat{x}(1-\psi)^{k-3}] \geq 0.$$

Thus ψ is unique.

Next, based on (3),

$$H(\hat{x}) = \hat{x} + (1-\lambda)\hat{x}(1-\hat{x}) \sum_k p_k [\hat{x}^{k-1} - (1-\hat{x})^{k-1}] \geq \hat{x}.$$

The inequality follows from the fact that $\hat{x}^{k-1} - (1-\hat{x})^{k-1} \geq 0$ when $\hat{x} \geq 1/2$. Combining with the earlier result that $\lim_{\psi \rightarrow 1} H(\psi) < 1$, we conclude that the unique ψ must be within $[\hat{x}, 1)$. Moreover, $H(\hat{x}) \geq \hat{x}$ means that the $H(\psi)$ curve crosses the 45-degree line from above. This property further implies that the steady-state ψ is globally stable, as $\psi_{T+1} = H(\psi_T) > \psi_T$ when $\psi_T < \psi$ and $\psi_{T+1} = H(\psi_T) < \psi_T$ for $\psi_T > \psi$.

Finally, to show the monotonicity of ψ in \hat{x} , let $\hat{x}_2 > \hat{x}_1 \geq 1/2$, and ψ_j be the corresponding steady-state ψ with \hat{x}_j . That is, $\psi_j = H(\hat{x}_j, \psi_j)$. By (3), we have

$$\frac{\partial H}{\partial \hat{x}} = 1 - (1-\lambda) \sum_k p_k [(\psi)^k + (1-\psi)^k] > 0,$$

as the term of summation is less than 1. This implies that $H(\hat{x}_2, \psi_1) > H(\hat{x}_1, \psi_1) = \psi_1$. Now consider the case with \hat{x}_2 . The fact that $H(\hat{x}_2, \psi_1) > \psi_1$ implies that at ψ_1 the $H(\psi)$ curve lies above the 45-degree line. Because the $H(\psi)$ curve crosses the 45-degree line from above, we must have $\psi_2 > \psi_1$. ■

Proof of Lemma 2.

Proof. Based on (3),

$$H(\hat{x}) = \hat{x} + (1 - \lambda)\hat{x}(1 - \hat{x}) \sum_k p_k [\hat{x}^{k-1} - (1 - \hat{x})^{k-1}]. \quad (22)$$

Part (i). Under the single-friend network, (22) becomes $H(\hat{x}) = \hat{x}$ with $p_1 = 1$, and thus $\psi = \hat{x}$. Under the infinite-friend network, when $k \rightarrow \infty$, both $\sum_k p_k \hat{x}^{k-1}$ and $\sum_k p_k (1 - \hat{x})^{k-1}$ go to 0. Therefore, again $H(\hat{x}) = \hat{x}$ and $\psi = \hat{x}$.

Part (ii). Consider any generic network. In (22), $\sum_k p_k [\hat{x}^{k-1} - (1 - \hat{x})^{k-1}] > 0$ when $\hat{x} > 1/2$ and $p_k > 0$ for some finite $k \geq 2$, which implies that $H(\hat{x}) > \hat{x}$. It means that $\psi > \hat{x}$, because ψ is unique given \hat{x} and the $H(\psi)$ curve crosses the 45-degree line from above.

Part (iii). Under the k -friend network, the steady-state equation (3) is written as

$$\psi = \hat{x} + (1 - \lambda)[(1 - \hat{x})(\psi)^k - \hat{x}(1 - \psi)^k] \equiv H(\psi, k). \quad (23)$$

Fixing \hat{x} , denote ψ_k as the solution to (23). For $k \geq 2$, to show that $\psi_{k+1} < \psi_k$, it is sufficient that $H(\psi_k, k+1) < \psi_k$; that is, under the $(k+1)$ -friend network, $H(\psi_k)$ lies below the 45-degree line (recall that at ψ_{k+1} , the $H(\psi, k+1)$ curve crosses the 45 degree line from above by Lemma 1).

$$\begin{aligned} H(\psi_k, k+1) &= \hat{x} + (1 - \lambda)[(1 - \hat{x})(\psi_k)^{k+1} - \hat{x}(1 - \psi_k)^{k+1}] \\ &= \hat{x} + (1 - \lambda)[(1 - \hat{x})(\psi_k)^k - \hat{x}(1 - \psi_k)^k] \\ &\quad - (1 - \lambda)\psi_k(1 - \psi_k)[(1 - \hat{x})(\psi_k)^{k-1} - \hat{x}(1 - \psi_k)^{k-1}] \\ &< \hat{x} + (1 - \lambda)[(1 - \hat{x})(\psi_k)^k - \hat{x}(1 - \psi_k)^k] = H(\psi_k, k) = \psi_k. \end{aligned}$$

The inequality holds for $k \geq 2$ because $(1 - \hat{x})(\psi_k)^{k-1} - \hat{x}(1 - \psi_k)^{k-1} > 0$ with $\psi_k > \hat{x} > 1/2$ based on part (ii). ■

Proof of Proposition 1.

Proof. We first show that a solution exists. With \hat{x} being the horizontal axis and ψ the vertical axis, the steady-state equation (3) defines a SS-curve and the pricing equation (6) defines a PE-curve. A solution is an intersection of these two curves. It is obvious that both curves are continuous. By (3), when $\hat{x} = 1/2$, we have $\psi = 1/2$. Thus $(\frac{1}{2}, \frac{1}{2})$ is the starting point of the SS-curve. By (6), when $\hat{x} = 1/2$, we have $\psi \geq 1/2$, because $\frac{d\psi}{d\hat{x}} > 0$ by Lemma 1. Therefore, the starting point of the PE-curve is weakly above that of the SS-curve. Next consider $\hat{x} = \frac{1}{2} + \frac{\Delta}{2t}$. By Lemma 2 on the SS-curve we have $\psi(\hat{x} = \frac{1}{2} + \frac{\Delta}{2t}) \geq \hat{x} = \frac{1}{2} + \frac{\Delta}{2t}$. By

(6), $\psi(\hat{x} = \frac{1}{2} + \frac{\Delta}{2t}) = 1/2$ on the PE-curve, because $\frac{d\psi}{d\hat{x}} > 0$. Therefore, at $\hat{x} = \frac{1}{2} + \frac{\Delta}{2t}$, the PE-curve is weakly below the SS-curve. By continuity, the two curves must intersect at some $\hat{x} \in [\frac{1}{2}, \frac{1}{2} + \frac{\Delta}{2t}]$, which is a solution. Because $\hat{x}_e \in [\frac{1}{2}, \frac{1}{2} + \frac{\Delta}{2t}]$, in the solution we must have $\hat{x}_e = 1/2$ when $\Delta = 0$, which implies that $\psi_e = 1/2$ as well.

Next we show the uniqueness of the solution. The pricing equation (6) can be more compactly written as

$$\hat{x} = \frac{1}{2} + \frac{\Delta}{2t} - (2\psi - 1) \frac{d\hat{x}}{d\psi}. \quad (24)$$

The derivative of the RHS of (24) with respect to ψ equals to

$$-[(2\psi - 1) \frac{d^2\hat{x}}{d\psi^2} + 2 \frac{d\hat{x}}{d\psi}],$$

which is strictly negative. To see this, by Lemma A1 in the Online Appendix, $\frac{d^2\hat{x}}{d\psi^2} \leq 0$ when $\psi \leq 1/2$, and $\frac{d^2\hat{x}}{d\psi^2} \geq 0$ when $\psi \geq 1/2$. Therefore, $(2\psi - 1) \frac{d^2\hat{x}}{d\psi^2} \geq 0$ for any $\psi \in [0, 1]$. Together with the fact that $\frac{d\hat{x}}{d\psi} > 0$, we have the desired result. In addition, the partial derivative of the RHS of (6) with respect to \hat{x} is negative. This means that the PE-curve is downward sloping. Note that the SS-curve is upward sloping because $\frac{d\psi}{d\hat{x}} > 0$. Thus, the two curves can have only one intersection; that is, the solution is unique.

Finally, we show the sufficiency of the first-order conditions by checking the second-order conditions. We will only prove the result for firm 1, as firm 2's situation is similar. In particular,

$$\frac{\partial^2 \pi_1}{\partial P_1^2} \propto -2 \frac{d\psi}{d\hat{x}} + \frac{P_1}{2t} \frac{d^2\psi}{d\hat{x}^2}.$$

Because $\frac{d\psi}{d\hat{x}} > 0$, and by Lemma A1 in the Online Appendix, $\frac{d^2\psi}{d\hat{x}^2} \leq 0$ when $\psi \geq 1/2$, we have $\frac{\partial^2 \pi_1}{\partial P_1^2} < 0$ when $\psi \geq 1/2$. When $\psi \leq 1/2$ (P_1 is relatively large), because $\frac{d^2\psi}{d\hat{x}^2} \geq 0$ by Lemma A1 in the Online Appendix, the sign of $\frac{\partial^2 \pi_1}{\partial P_1^2}$ is indeterminate. To ensure $\frac{\partial^2 \pi_1}{\partial P_1^2} \leq 0$, $|\frac{d^2\psi}{d\hat{x}^2}|$ has to be small enough. By Lemma A1 in the Online Appendix, $\lim_{\lambda \rightarrow 1} \frac{d^2\psi}{d\hat{x}^2} = 0$ and $\frac{d^2\psi}{d\hat{x}^2} \rightarrow 0$ under either well-connected networks or the single-friend network. Therefore, the second-order condition $\frac{\partial^2 \pi_1}{\partial P_1^2} \leq 0$ is satisfied if either λ is large enough or the network is either well-connected or the single-friend network. ■

Proof of Proposition 2.

Proof. Part (i). By (7), $\frac{d\psi}{d\hat{x}} = 1$ under both the single-friend network and the infinite-friend network. Then $P^e = t$ immediately follows. Now consider any generic network, with $p_l > 0$

for some finite $l \geq 2$. Under these networks, it is obvious that

$$\sum_k k p_k \left(\frac{1}{2}\right)^{k-1} > \sum_k p_k \left(\frac{1}{2}\right)^{k-1}.$$

By (7), this implies that $\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2} > 1$. Therefore, $P^e < t$.

Part (ii). Under the k -friend networks, by (7) we have

$$\left[\frac{d\psi}{d\hat{x}}(k+1) - \frac{d\psi}{d\hat{x}}(k)\right]|_{\hat{x}=1/2} \propto (2-k)\left(\frac{1}{2}\right)^k - (1-\lambda)\left(\frac{1}{2}\right)^{2k-1},$$

which is positive if $k = 1$, but is negative for any $k \geq 2$. The statement in the proposition immediately follows. ■

Proof of Proposition 3.

Proof. Part (i). It is sufficient to show that $[\frac{d\psi}{d\hat{x}}(p'_k) - \frac{d\psi}{d\hat{x}}(p''_k)]|_{\hat{x}=1/2} > 0$. By (7), $[\frac{d\psi}{d\hat{x}}(p'_k) - \frac{d\psi}{d\hat{x}}(p''_k)]|_{\hat{x}=1/2}$ has the same sign as

$$\sum_k (k-1)(p'_k - p''_k)\left(\frac{1}{2}\right)^{k-1} + (1-\lambda)\left[\sum_k p'_k \left(\frac{1}{2}\right)^{k-1} \sum_k k p''_k \left(\frac{1}{2}\right)^{k-1} - \sum_k p''_k \left(\frac{1}{2}\right)^{k-1} \sum_k k p'_k \left(\frac{1}{2}\right)^{k-1}\right].$$

Because $(\frac{1}{2})^{k-1}$ is decreasing in k and $\{p''_k\}$ FOSD $\{p'_k\}$, $A \equiv \sum_k p''_k (\frac{1}{2})^{k-1} < \sum_k p'_k (\frac{1}{2})^{k-1} \equiv B$. The term $(k-1)(\frac{1}{2})^{k-1}$ is constant when k changes from 2 to 3, and decreases in k for $k \geq 3$. Because $\{p''_k\}$ FOSD $\{p'_k\}$ and $p''_1 = p'_1$, relative to $\{p'_k\}$, $\{p''_k\}$ puts higher probabilities on $k \geq 3$. Therefore, $Z \equiv \sum_k (k-1)(p'_k - p''_k)(\frac{1}{2})^{k-1} > 0$. Similarly, $C \equiv \sum_k k p''_k (\frac{1}{2})^{k-1} < \sum_k k p'_k (\frac{1}{2})^{k-1} \equiv D$, because the term $k(\frac{1}{2})^{k-1}$ is constant when k changes from 1 to 2, and is decreasing in k for $k \geq 2$. Note that A, B, C , and D are all smaller than 1. Moreover,

$$(A+D) - (B+C) = \sum_k (k-1)(p'_k - p''_k)\left(\frac{1}{2}\right)^{k-1} = Z > 0.$$

Using more compact notations, we have

$$\left[\frac{d\psi}{d\hat{x}}(p'_k) - \frac{d\psi}{d\hat{x}}(p''_k)\right]|_{\hat{x}=1/2} \propto Z + (1-\lambda)(BC - AD).$$

If $BC \geq AD$, then we get the desired result that $[\frac{d\psi}{d\hat{x}}(p'_k) - \frac{d\psi}{d\hat{x}}(p''_k)]|_{\hat{x}=1/2} > 0$. Next consider

the case that $BC < AD$. In particular,

$$Z + (1 - \lambda)(BC - AD) > Z + BC - AD > B(C + Z) - AD > 0.$$

The first inequality holds because $BC < AD$. The second inequality uses the fact that $B < 1$. The last inequality holds because $B + C + Z = A + D$, $A < B < D$, and $A < C + Z < D$. Therefore, again $[\frac{d\psi}{d\hat{x}}(p'_k) - \frac{d\psi}{d\hat{x}}(p''_k)]|_{\hat{x}=1/2} > 0$.

Part (ii). By (7),

$$\frac{\partial(\frac{d\psi}{d\hat{x}}|_{\hat{x}=1/2})}{\partial\lambda} \propto \sum_k (1 - k)p_k(\frac{1}{2})^{k-1},$$

which is negative if there is a finite $k \geq 2$ such that $p_k > 0$. Therefore, the equilibrium price increases in λ . ■

Proof of Proposition 4.

Proof. Because $(\frac{1}{2})^{k+1}$ is decreasing k , a FOSD change in $\{p_k\}$ reduces $\sum_k p_k(\frac{1}{2})^{k+1}$. By (8), this implies that W increases.

Next consider consumer surplus CS . For the k -friend networks, we can compute the difference in CS as k increases by 1 based on equation (9):

$$CS(k) - CS(k+1) \propto (\frac{1}{2})^{k+2}[(4k - 9) + 11(1 - \lambda)(\frac{1}{2})^k - (1 - \lambda)^2(\frac{1}{2})^{2k-1}]. \quad (25)$$

It can be verified that (25) is negative when $k = 1$. Therefore, $CS(k = 2) > CS(k = 1)$. This proves part (i). For part (ii), it can be verified that (25) is positive when $k \geq 3$. Therefore, CS is decreasing in k when $k \geq 3$. Finally, for part (iii), when $k = 2$ the term in the bracket in (25) becomes

$$-1 + \frac{11}{4}(1 - \lambda) - \frac{(1 - \lambda)^2}{8},$$

which is positive if $\lambda \leq \sqrt{113} - 10 \simeq 0.63$ and negative otherwise. The result immediately follows. ■

Proof of Lemma 3.

Proof. Parts (i) and (ii). The proof is similar to that of Lemma 1. Define the LHS of (10) and (11) as $H_L(\psi_L)$ and $H_R(\psi_R)$, respectively. Note that both $H_L(\cdot)$ and $H_R(\cdot)$ are continuous functions. Given ψ , the steady-state ψ_L satisfies $\psi_L = H_L(\psi_L)$ and ψ_R satisfies $\psi_R = H_R(\psi_R)$. We first show that for any given ψ , there is a unique $\psi_L \in (\psi, 1)$. It can be verified that $H_L(\psi) = 1 - (1 - \lambda)\sum_k p_k(1 - \psi)^k \geq 1 - (1 - \lambda)(1 - \psi) > \psi$, and $H_L(1) =$

$1 - (1 - \lambda) \sum_k p_k [(1 - \alpha)(1 - \psi)]^k < 1$. Thus the continuity of $H_L(\cdot)$ implies the existence of a $\psi_L \in (\psi, 1)$ which satisfies $\psi_L = H_L(\psi_L)$. To show the uniqueness, it is sufficient that $\frac{\partial^3 H_L}{\partial \psi^3} > 0$, which implies that the H_L -curve crosses the 45 degree line at most once. It is straightforward to check that $\frac{\partial^3 H_L}{\partial \psi^3} > 0$ holds, thus we have the uniqueness of ψ_L .

Next we show that ψ_L is increasing in ψ . Notice that $H_L(\psi) > \psi$ also implies that the H_L -curve crosses the 45 degree line from above. As ψ increases, the H_L -curve shifts up, which means that ψ_L increases, or $\frac{\partial \psi_L}{\partial \psi} > 0$. To show that ψ_L is increasing in α , consider $\alpha'' > \alpha'$, and denote the corresponding steady-state ψ_L as ψ'_L and ψ''_L , respectively. Because $\psi'_L > \psi$ and $\alpha'' > \alpha'$, by (10) we have $\psi'_L = H_L(\psi'_L, \alpha') < H_L(\psi'_L, \alpha'')$. Given that the H_L -curve crosses the 45 degree line from above, $\psi''_L > \psi'_L$ must hold.

Following similar steps, we can show the results regarding ψ_R . First, $H_R(0) > 0$ and $H_R(\psi) = (1 - \lambda) \sum_k p_k \psi^k < \psi$. Second, $\frac{\partial^3 H_R}{\partial \psi^3} > 0$. These properties ensure a unique $\psi_R \in (0, \psi)$ satisfying $\psi_R = H_R(\psi_R)$, and that the H_R -curve crosses the 45 degree line from above. As ψ increases, the H_R -curve shifts up, which means that ψ_R increases, or $\frac{\partial \psi_R}{\partial \psi} > 0$. Finally, consider $\alpha'' > \alpha'$. As $\psi'_R < \psi$, by (11) $\alpha'' > \alpha'$ implies that $\psi'_R = H_R(\psi'_R, \alpha') > H_R(\psi'_R, \alpha'')$. Given that the H_R -curve crosses the 45 degree line from above, $\psi''_R < \psi'_R$ must hold.

Part (iii). Denote $\phi_L \equiv 1 - \psi_L$. By (10) and (11), we have

$$\phi_L = (1 - \lambda) \sum_k p_k [\phi_L + (1 - \alpha)(1 - \psi - \phi_L)]^k, \quad (26)$$

$$\psi_R = (1 - \lambda) \sum_k p_k [\psi_R + (1 - \alpha)(\psi - \psi_R)]^k. \quad (27)$$

Define the RHS of (26) as $G_L(\phi_L)$, and thus $G_L(\phi_L) = \phi_L$. Recall that the RHS of (27) is $H_R(\psi_R)$. As $\psi \geq 1/2$, comparing (26) and (27), we have $H_R(y) \geq G_L(y)$ for any $y \leq \psi$. Therefore, $\phi_L = G_L(\phi_L) \leq H_R(\phi_L)$. The fact that $\phi_L \leq H_R(\phi_L)$ implies that $\psi_R \geq \phi_L = 1 - \psi_L$, as the H_R -curve crosses the 45 degree line from above. ■

Proof of Lemma 4.

Proof. Part (i). Denote $\psi_L(\psi)$ and $\psi_R(\psi)$ as the functions of the steady-state ψ_L and ψ_R when ψ changes. Then $H(\psi_L(\psi), \psi_R(\psi)) \equiv \hat{x}\psi_L(\psi) + (1 - \hat{x})\psi_R(\psi)$, and the steady-state ψ satisfies $\psi = H(\psi_L(\psi), \psi_R(\psi))$. For the existence of a steady state, it suffices to show that (a) $H(\psi_L(\psi), \psi_R(\psi))$ is continuous, (b) $\lim_{\psi \rightarrow 0} H(\psi_L(\psi), \psi_R(\psi)) \geq 0$, and (c) $\lim_{\psi \rightarrow 1} H(\psi_L(\psi), \psi_R(\psi)) \leq 1$. Note that (a) holds because both $\psi_L(\psi)$ and $\psi_R(\psi)$ are continuous, and both (b) and (c) are satisfied because, by (10) and (11), $\psi_L \in [0, 1]$ and $\psi_R \in [0, 1]$. Therefore, the existence of a steady-state ψ is ensured. For the uniqueness of ψ , it is enough to show that (d) $\frac{d^3}{d\psi^3} H(\psi_L(\psi), \psi_R(\psi)) \geq 0$, which implies that $H(\psi_L(\psi), \psi_R(\psi))$ crosses the

45-degree line at most once. Actually, combined with (b) and (c), property (d) also implies that the $H(\psi)$ curve crosses the 45-degree line from above at the steady-state ψ .

To show property (d), it is enough to show that $\frac{\partial^3 \psi_L}{\partial \psi^3} \geq 0$ and $\frac{\partial^3 \psi_R}{\partial \psi^3} \geq 0$, because $\frac{d^3}{d\psi^3} H(\psi_L(\psi), \psi_R(\psi)) = \hat{x} \frac{\partial^3 \psi_L}{\partial \psi^3} + (1 - \hat{x}) \frac{\partial^3 \psi_R}{\partial \psi^3}$. For that purpose, define $f(z) \equiv \sum_k p_k z^k$, $z_L \equiv 1 - (1 - \alpha)\psi - \alpha\psi_L$, and $z_R \equiv (1 - \alpha)\psi + \alpha\psi_R$. It can be readily verified that $\frac{\partial^n}{\partial z^n} f(z) \geq 0$ for all n . Moreover, $\frac{\partial z_L}{\partial \psi} = -(1 - \alpha + \alpha \frac{\partial \psi_L}{\partial \psi})$ and $\frac{\partial z_R}{\partial \psi} = (1 - \alpha + \alpha \frac{\partial \psi_R}{\partial \psi})$. Differentiating (10) with respect to ψ yields

$$\frac{\partial \psi_L}{\partial \psi} = \frac{(1 - \lambda)(1 - \alpha)f'(z_L)}{1 - (1 - \lambda)\alpha f'(z_L)}.$$

The numerator of the above expression is positive. By Lemma 3, $\frac{\partial \psi_L}{\partial \psi} > 0$. Thus the denominator, $1 - (1 - \lambda)\alpha f'(z_L)$, is also positive. That $\frac{\partial \psi_L}{\partial \psi} > 0$ also implies that $\frac{\partial z_L}{\partial \psi} < 0$. Differentiating (10) repeatedly, we get

$$\begin{aligned} \frac{\partial^2 \psi_L}{\partial \psi^2} &= \frac{-(1 - \lambda)f''(z_L)(\frac{\partial z_L}{\partial \psi})^2}{1 - (1 - \lambda)\alpha f'(z_L)}, \\ \frac{\partial^3 \psi_L}{\partial \psi^3} &= \frac{(1 - \lambda)[-f'''(z_L)(\frac{\partial z_L}{\partial \psi})^3 + 3\alpha f''(z_L)\frac{\partial z_L}{\partial \psi}\frac{\partial^2 \psi_L}{\partial \psi^2}]}{1 - (1 - \lambda)\alpha f'(z_L)}. \end{aligned}$$

Note that $\frac{\partial^2 \psi_L}{\partial \psi^2} < 0$, as the numerator is negative. On the other hand, $\frac{\partial^3 \psi_L}{\partial \psi^3} > 0$, because the numerator is positive as $\frac{\partial z_L}{\partial \psi} < 0$ and $\frac{\partial^2 \psi_L}{\partial \psi^2} < 0$.

Similarly, regarding ψ_R we have

$$\begin{aligned} \frac{\partial \psi_R}{\partial \psi} &= \frac{(1 - \lambda)(1 - \alpha)f'(z_R)}{1 - (1 - \lambda)\alpha f'(z_R)} > 0, \\ \frac{\partial^2 \psi_R}{\partial \psi^2} &= \frac{(1 - \lambda)f''(z_R)(\frac{\partial z_R}{\partial \psi})^2}{1 - (1 - \lambda)\alpha f'(z_R)}, \\ \frac{\partial^3 \psi_R}{\partial \psi^3} &= \frac{(1 - \lambda)[f'''(z_R)(\frac{\partial z_R}{\partial \psi})^3 + 3\alpha f''(z_R)\frac{\partial z_R}{\partial \psi}\frac{\partial^2 \psi_R}{\partial \psi^2}]}{1 - (1 - \lambda)\alpha f'(z_R)}. \end{aligned}$$

The fact that $\frac{\partial \psi_R}{\partial \psi} > 0$ implies that $1 - (1 - \lambda)\alpha f'(z_R) > 0$ and $\frac{\partial z_R}{\partial \psi} > 0$. Then it is straightforward to check that $\frac{\partial^2 \psi_R}{\partial \psi^2} > 0$ and $\frac{\partial^3 \psi_R}{\partial \psi^3} > 0$. This completes the proof that the steady-state ψ is unique.

Next, we show that $\psi \geq \hat{x}$. By (12),

$$H(\psi) = \hat{x} + [(1 - \hat{x})\psi_R(\psi) - \hat{x}(1 - \psi_L(\psi))].$$

By earlier results, it suffices to show $H(\hat{x}) \geq \hat{x}$, which is equivalent to $(1 - \hat{x})\psi_R(\hat{x}) - \hat{x}(1 - \psi_L(\hat{x})) \geq 0$. Somewhat abusing the notations, let $z_L = 1 - (1 - \alpha)\hat{x} - \alpha\psi_L$ and $z_R = (1 - \alpha)\hat{x} + \alpha\psi_R$. When $\psi = \hat{x}$, (10) becomes $1 - \psi_L = (1 - \lambda)\sum_k p_k z_L^k$ and (11) becomes $\psi_R = (1 - \lambda)\sum_k p_k z_R^k$. By Lemma 3, $1 - \psi_L \leq \psi_R$ when $\psi = \hat{x} \geq 1/2$, thus we have $z_L \leq z_R$.

Note that

$$(1 - \hat{x})z_R - \hat{x}z_L = \alpha[(1 - \hat{x})\psi_R(\hat{x}) - \hat{x}(1 - \psi_L(\hat{x}))].$$

Thus it suffices to show that $(1 - \hat{x})z_R - \hat{x}z_L \geq 0$ or $\frac{1 - \hat{x}}{\hat{x}} \frac{z_R}{z_L} - 1 \geq 0$. Again by (10) and (11),

$$(1 - \hat{x})z_R - \hat{x}z_L = \alpha(1 - \lambda) \sum_k p_k [(1 - \hat{x})z_R^k - \hat{x}z_L^k],$$

which leads to

$$\frac{1 - \hat{x}}{\hat{x}} \frac{z_R}{z_L} - 1 = \alpha(1 - \lambda) \sum_k p_k z_L^{k-1} \left[\frac{1 - \hat{x}}{\hat{x}} \left(\frac{z_R}{z_L} \right)^k - 1 \right]. \quad (28)$$

Now suppose $\frac{1 - \hat{x}}{\hat{x}} \frac{z_R}{z_L} - 1 < 0$. Because $z_L \leq z_R$, $\frac{1 - \hat{x}}{\hat{x}} \left(\frac{z_R}{z_L} \right)^k - 1 \geq \frac{1 - \hat{x}}{\hat{x}} \frac{z_R}{z_L} - 1$ for all k . Given that $\alpha(1 - \lambda)z_L^{k-1} < 1$ for all k and the LHS of (28) is strictly negative, for each k , the term in the RHS of (28) is either positive or strictly less negative than the LHS. Thus, (28) cannot hold with equality, leading to a contradiction. Therefore, we must have $\frac{1 - \hat{x}}{\hat{x}} \frac{z_R}{z_L} - 1 \geq 0$.

Part (ii). The result that $\psi_R < \psi < \psi_L$ directly follows from Lemma 3. Finally, to show that ψ is increasing in \hat{x} , consider any $\hat{x}' > \hat{x}'' \geq 1/2$, and denote the corresponding steady-state ψ as ψ'_S and ψ''_S , respectively. Note that $H(\psi''_S; \hat{x}'') = \hat{x}''\psi_L(\psi''_S) + (1 - \hat{x}'')\psi_R(\psi''_S)$, and $\psi_L(\psi''_S) > \psi_R(\psi''_S)$ by Lemma 3. Therefore, $H(\psi'_S; \hat{x}') = \hat{x}'\psi_L(\psi'_S) + (1 - \hat{x}')\psi_R(\psi'_S) > H(\psi''_S; \hat{x}'') = \psi''_S$. Combined with the fact that the $H(\psi)$ curve crosses the 45-degree line from above, this implies $\psi'_S > \psi''_S$. This proves that ψ strictly increases in \hat{x} . ■

Proof of Lemma 5.

Proof. Part (i). First consider the single-friend network. With $p_1 = 1$, (13) becomes

$$\psi - \hat{x} = \frac{(1 - \lambda)(1 - \alpha)}{1 - (1 - \lambda)\alpha} (\psi - \hat{x}),$$

which implies that $\psi = \hat{x}$.

Under the infinite-friend network, by (10) and (11), we have $\psi_L = 1$ and $\psi_R = 0$. Then $\psi = \hat{x}$ based on (12). Finally, consider the case of any generic network with $\alpha = 1$. By (10) and (11), again we have $\psi_L = 1$ and $\psi_R = 0$. Therefore, $\psi = \hat{x}$.

Part (ii). To show $\psi > \hat{x}$, we follow similar steps as in the proof of part (ii) of Lemma 4, where the weak inequality is proved. With a generic network and $\hat{x} > 1/2$, following part (iii)

of Lemma 3, we can show that $\psi_R > 1 - \psi_L$ if $\psi > 1/2$, which leads to $\frac{z_R}{z_L} > 1$. Then in part (ii) of Lemma 4, $\frac{1-\hat{x}}{\hat{x}}(\frac{z_R}{z_L})^k - 1 > \frac{1-\hat{x}}{\hat{x}}\frac{z_R}{z_L} - 1$ for all k . Now suppose $\frac{1-\hat{x}}{\hat{x}}\frac{z_R}{z_L} - 1 = 0$. Then $\frac{1-\hat{x}}{\hat{x}}(\frac{z_R}{z_L})^k - 1 > 0$ for all $k \geq 2$. Therefore, the RHS of (28) is strictly positive, contradicting the equality. Therefore, $H(\hat{x}) > \hat{x}$ and $\psi > \hat{x}$.

Finally, we show that ψ decreases in α . By (10) and (11),

$$\begin{aligned}\frac{\partial\psi_L}{\partial\alpha} &= -(1-\lambda)f'(z_L)\left[-(1-\alpha)\frac{\partial\psi}{\partial\alpha} + \psi - \psi_L - \alpha\frac{\partial\psi_L}{\partial\alpha}\right] \\ &= \frac{(1-\lambda)f'(z_L)}{1-\alpha(1-\lambda)f'(z_L)}\left[(1-\alpha)\frac{\partial\psi}{\partial\alpha} - \psi + \psi_L\right] \\ &= \frac{\partial\psi_L}{\partial\psi}\frac{\partial\psi}{\partial\alpha} + \frac{(1-\lambda)f'(z_L)}{1-\alpha(1-\lambda)f'(z_L)}(1-\hat{x})(\psi_L - \psi_R).\end{aligned}$$

The last equality follows from $\psi_L - \psi = (1-\hat{x})(\psi_L - \psi_R)$. Similarly,

$$\begin{aligned}\frac{\partial\psi_R}{\partial\alpha} &= (1-\lambda)f'(z_R)\left[(1-\alpha)\frac{\partial\psi}{\partial\alpha} - \psi + \psi_R + \alpha\frac{\partial\psi_R}{\partial\alpha}\right] \\ &= \frac{(1-\lambda)f'(z_R)}{1-\alpha(1-\lambda)f'(z_R)}\left[(1-\alpha)\frac{\partial\psi}{\partial\alpha} - \psi + \psi_R\right] \\ &= \frac{\partial\psi_R}{\partial\psi}\frac{\partial\psi}{\partial\alpha} - \frac{(1-\lambda)f'(z_R)}{1-\alpha(1-\lambda)f'(z_R)}\hat{x}(\psi_L - \psi_R).\end{aligned}$$

The last equality follows from $\psi_R - \psi = -\hat{x}(\psi_L - \psi_R)$. Combine the above results,

$$\begin{aligned}\frac{\partial\psi}{\partial\alpha} &= \hat{x}\frac{\partial\psi_L}{\partial\alpha} + (1-\hat{x})\frac{\partial\psi_R}{\partial\alpha} \\ &= \left[\hat{x}\frac{\partial\psi_L}{\partial\psi} + (1-\hat{x})\frac{\partial\psi_R}{\partial\psi}\right]\frac{\partial\psi}{\partial\alpha} \\ &\quad + \hat{x}(1-\hat{x})(1-\lambda)(\psi_L - \psi_R)\left[\frac{f'(z_L)}{1-\alpha(1-\lambda)f'(z_L)} - \frac{f'(z_R)}{1-\alpha(1-\lambda)f'(z_R)}\right] \\ &\propto \frac{\frac{f'(z_L)}{1-\alpha(1-\lambda)f'(z_L)} - \frac{f'(z_R)}{1-\alpha(1-\lambda)f'(z_R)}}{1 - \left[\hat{x}\frac{\partial\psi_L}{\partial\psi} + (1-\hat{x})\frac{\partial\psi_R}{\partial\psi}\right]}, \text{ because } \psi_L > \psi_R \\ &\propto \frac{f'(z_L)}{1-\alpha(1-\lambda)f'(z_L^*)} - \frac{f'(z_R)}{1-\alpha(1-\lambda)f'(z_R^*)}, \text{ because } \frac{\partial\psi}{\partial\hat{x}} > 0 \\ &< 0.\end{aligned}$$

The last inequality holds because $f'(z)$ is increasing in z and $z_L < z_R$, as shown earlier. ■

Proof of Proposition 5.

Proof. We first show the existence of a solution for $\Delta \in [0, t]$. With \hat{x} being the horizontal axis and ψ the vertical axis, a solution is an intersection of the SS-curve defined by the steady-state equations (10)-(12) and the PE-curve defined by the pricing equation (14). Both curves are continuous. We can verify that $(\frac{1}{2}, \frac{1}{2})$ is the starting point of the SS-curve. For the PE-curve, based on (14) and the fact that $\frac{d\psi}{d\hat{x}} > 0$, $\psi \geq 1/2$ when $\hat{x} = 1/2$. Therefore, the starting point of the PE-curve is weakly above the SS-curve. Now consider the ending point at $\hat{x} = \frac{1}{2} + \frac{\Delta}{2t}$. By lemma 4, on the SS-curve $\psi(\frac{1}{2} + \frac{\Delta}{2t}) \geq \frac{1}{2} + \frac{\Delta}{2t}$. In addition, by the pricing equation (14), on the PE-curve $\psi = \frac{1}{2}$ when $\hat{x} = \frac{1}{2} + \frac{\Delta}{2t}$, as $\frac{d\psi}{d\hat{x}} > 0$. Therefore, the PE-curve is weakly below the SS-curve at the ending point. By continuity, the two curves must intersect at some $\hat{x} \in [\frac{1}{2}, \frac{1}{2} + \frac{\Delta}{2t}]$, which is a solution. As $\hat{x}_e \in [\frac{1}{2}, \frac{1}{2} + \frac{\Delta}{2t}]$, we must have $\hat{x}_e = 1/2$ when $\Delta = 0$, which implies that $\psi_e = 1/2$ as well.

Next, the uniqueness of candidate equilibrium follows from the same proof as in Proposition 1. That the PE-curve is downward sloping follows from the pricing equation (14) and Lemma A2 in the Online Appendix. The SS-curve is upward sloping by Lemma 4. Thus, the two curves can only have one intersection and the candidate equilibrium is unique.

Finally, following an argument similar to the proof of Proposition 1, by Lemma A2 in the Online Appendix the second-order conditions are satisfied if either λ or α is large enough, or the network is well-connected or the single-friend network. Thus, the sufficiency of the first-order conditions is guaranteed. ■

Proof of Proposition 6.

Proof. Part (i). Following part (i) of Lemma 5, under both networks $\psi = \hat{x}$. And thus $\frac{d\psi}{d\hat{x}} = 1$ at any \hat{x} . The results immediately follow.

Part (ii). It is sufficient to show that $\frac{d\psi}{d\hat{x}}|_{\hat{x}=\frac{1}{2}}$ is decreasing in α . Let $z \equiv (1 - \lambda) \sum_k k p_k [(1 - \alpha)/2 + \alpha \psi_R]^{k-1}$. Then (16) can be written compactly as

$$\frac{d\psi}{d\hat{x}}|_{\hat{x}=\frac{1}{2}} = (1 - 2\psi_R) \frac{1 - \alpha z}{1 - z}. \quad (29)$$

By definition,

$$\frac{\partial z}{\partial \alpha} = (1 - \lambda) \sum_k k(k-1) p_k [(1 - \alpha)/2 + \alpha \psi_R]^{k-2} (\psi_R - \frac{1}{2} + \alpha \frac{\partial \psi_R}{\partial \alpha}) < 0,$$

as by Lemma 3, $\psi_R < \psi = \frac{1}{2}$ and $\frac{\partial \psi_R}{\partial \alpha} < 0$. Now differentiating (29) with respect to α , we get

$$\begin{aligned} \frac{\partial^2 \psi}{\partial \hat{x} \partial \alpha} \Big|_{\hat{x}=\frac{1}{2}} &\propto -2 \frac{\partial \psi_R}{\partial \alpha} (1 - \alpha z)(1 - z) + (1 - 2\psi_R)[-z(1 - z) + (1 - \alpha) \frac{\partial z}{\partial \alpha}] \\ &= -2z(\psi_R - \frac{1}{2})(1 - z) + (1 - 2\psi_R)[-z(1 - z) + (1 - \alpha) \frac{\partial z}{\partial \alpha}] \\ &= (1 - 2\psi_R)(1 - \alpha) \frac{\partial z}{\partial \alpha} < 0. \end{aligned}$$

In the first equality above we used the result that $\frac{\partial \psi_R}{\partial \alpha} = \frac{z(\psi_R - \frac{1}{2})}{1 - \alpha z}$, which can be derived from (11). Therefore, $\frac{d\psi}{d\hat{x}} \Big|_{\hat{x}=\frac{1}{2}}$ is decreasing in α . ■

Proof of Proposition 7.

Proof. Part (i). Because ψ_R strictly decreases in α (Lemma 3), by (17) W strictly increases in α .

Part (ii). By Proposition 6, under the extreme networks $\frac{d\psi}{d\hat{x}} = 1$ for all α . The result immediately follows part (i).

Part (iii). Using the notations and results in the proof of Proposition 6, and let $y \equiv (1 - \alpha)/2 + \alpha\psi_R$, from (18) we compute

$$\begin{aligned} \frac{\partial CS}{\partial \alpha} &\propto (\psi_R - \frac{1}{2})(1 - \alpha z) \frac{\partial \psi_R}{\partial \alpha} + \frac{1 - z}{\psi_R - \frac{1}{2}} \frac{\partial \psi_R}{\partial \alpha} + \frac{-z(1 - z) + (1 - \alpha) \frac{\partial z}{\partial \alpha}}{1 - \alpha z} \\ &\propto (\psi_R - \frac{1}{2})^2 (1 - \alpha z) z + (1 - \alpha)(1 - \lambda) \sum_k p_k k(k - 1) y^{k-2} (\psi_R - \frac{1}{2} + \alpha \frac{\partial \psi_R}{\partial \alpha}) \\ &\propto \frac{1}{2} (1 - 2\psi_R)(1 - \alpha z)^2 z - (1 - \alpha)(1 - \lambda) \sum_k p_k k(k - 1) y^{k-2}. \end{aligned} \quad (30)$$

In the derivation we used the fact that $\psi_R < 1/2$.

To determine the sign of (30), define z' as follows and recall z :

$$\begin{aligned} z' &\equiv (1 - \lambda) \sum_k p_k k(k - 1) y^{k-2} = (1 - \lambda)[2p_2 + 6p_3 y + \dots], \\ z &= (1 - \lambda)[p_1 + 2p_2 y + 3p_3 y^2 + \dots]. \end{aligned}$$

Note that $y < 1/2$ because $\psi_R < 1/2$. Suppose $p_2 \geq p_1$. Because $y < 1/2$, $\frac{z' - z}{1 - \lambda} \geq 2p_2(1 - y) - p_1 > p_2 - p_1 \geq 0$; that is, $z' \geq z$. Then (30) is less than $\frac{1}{2}z' - (1 - \alpha)z'$, which is negative if $\alpha \leq 1/2$. Next consider the case that $p_1 = 0$. Now $y \leq 1/2$ implies that $z' \geq 2z$. Then (30) is less than $\frac{1}{2}z' - 2(1 - \alpha)z'$, which is negative if $\alpha \leq 3/4$. ■

Proof of Proposition 10.

Proof. Part (i). By (19), it is obvious that P^* is decreasing in δ .

Part (ii). Under the k -friend networks, by (19) we can calculate that $P^*|_{k+1} - P^*|_k \propto -1 + \delta\Phi(k)$, where

$$\Phi(k) = (k - 1) + (1 - \lambda)\left(\frac{1}{2}\right)^{k-1}.$$

When $k = 1$, $\Phi(k) = 1 - \lambda$ and hence $P^*|_2 - P^*|_1 < 0$. When k is vary large, $P^*|_{k+1} - P^*|_k > 0$ as long as $\delta > 0$. We can also verify that $\Phi(k + 1) - \Phi(k) = 1 - (1 - \lambda)\left(\frac{1}{2}\right)^k > 0$ for any $k \geq 1$. That is, $\Phi(k)$ is monotonically increasing in k . Then, there exists a $\hat{k} \geq 2$ such that $P^*|_{k+1} - P^*|_k$ is negative when $k \leq \hat{k}$ and is positive when $k \geq \hat{k}$. Moreover, because $P^*|_{k+1} - P^*|_k$ is increasing in δ , the cutoff \hat{k} decreases in δ . It can be also verified that $\hat{k} = 2$ when $\delta = 1$ and $\hat{k} \rightarrow \infty$. ■

Proof of Proposition 11.

Proof. Part (i). By (20), $\frac{d\psi_T}{dP_{1,T}} = \frac{d(1-\psi_T)}{dP_{2,T}} = -\frac{1}{2t}\phi_{F,T}$. Sometimes we write $\phi_{F,T}$ as $\phi_{F,T}(\psi_{T-1})$, emphasizing that it is a function of ψ_{T-1} . First, we solve the pricing game in period 2, given ψ_1 . Firm 1's and firm 2's profits in period 2 are $\Pi_{1,2} = P_{1,2}\psi_2$ and $\Pi_{2,2} = P_{2,2}(1 - \psi_2)$, respectively. The first-order conditions yield

$$\begin{aligned} (2P_{1,2} - P_{2,2} - t)\phi_{F,2} &= 2t(1 - \lambda) \sum_k p_k \psi_1^k, \\ (2P_{2,2} - P_{1,2} - t)\phi_{F,2} &= 2t(1 - \lambda) \sum_k p_k (1 - \psi_1)^k. \end{aligned}$$

By the above equations, we get the equilibrium prices:

$$P_{1,2} = t \frac{1 - \frac{1}{3}(1 - \lambda) \sum_k p_k [(1 - \psi_1)^k - \psi_1^k]}{\phi_{F,2}(\psi_1)}, \quad (31)$$

$$P_{2,2} = t \frac{1 - \frac{1}{3}(1 - \lambda) \sum_k p_k [\psi_1^k - (1 - \psi_1)^k]}{\phi_{F,2}(\psi_1)}. \quad (32)$$

Then we can compute firm 1's equilibrium profit in period 2 as a function of ψ_1 :

$$\Pi_{1,2}(\psi_1) = \frac{t}{18\phi_{F,2}(\psi_1)} \left[2 + \phi_{F,2}(\psi_1) + 2(1 - \lambda) \sum_k p_k \psi_1^k \right]^2.$$

In the symmetric equilibrium, $\psi_1 = 1/2$. This leads to $\phi_{F,2} = 1 - (1 - \lambda) \sum_k p_k \left(\frac{1}{2}\right)^{k-1}$ and $\frac{\partial \phi_{F,2}}{\partial \psi_1} |_{\psi_1=1/2} = 0$. Moreover, by (31) and (32), we have $P_{1,2} = P_{2,2} = P_2^*$, where P_2^* is given by

(21). In addition,

$$\frac{d\Pi_{1,2}(\psi_1)}{d\psi_1}\Big|_{\psi_1=1/2} = \frac{2t}{3} \frac{(1-\lambda) \sum_k p_k k (\frac{1}{2})^{k-1}}{1 - (1-\lambda) \sum_k p_k (\frac{1}{2})^{k-1}}.$$

Next, we solve the pricing game in period 1. Suppose firm 2 chooses P_1^* in period 1. Firm 1 chooses $P_{1,1}$ to maximize its discounted profit $\Pi_1 = P_{1,1}\psi_1 + \delta\Pi_{1,2}(\psi_1)$. The first-order condition yields

$$2P_{1,1} - P_1^* - t + \delta \frac{d\Pi_{1,2}}{d\psi_1} = \frac{2t(1-\lambda) \sum_k p_k \psi_0^k}{\phi_{F,1}(\psi_0)}.$$

Using the symmetry $P_{1,1} = P_1^*$, and $\psi_0 = 1/2$, we get the expression of P_1^* in (21). Finally, by (21), it is obvious that $P_1^* < P_2^*$.

Part (ii). By (19) and (21), it is clear that $P_1^* > P^*$. Actually, if we define $\tilde{\delta} \equiv \frac{2}{3}\delta$, then $P_1^*(\delta) = P^*(\tilde{\delta})$. Therefore, we can apply part (ii) of Proposition 10: there is a cutoff integer \tilde{k} such that P_1^* is decreasing in k when $k \leq \tilde{k}$ and increasing in k when $k > \tilde{k}$. As \tilde{k} is decreasing in δ , $\tilde{\delta} < \delta$ implies that $\tilde{k} > \hat{k}$. ■

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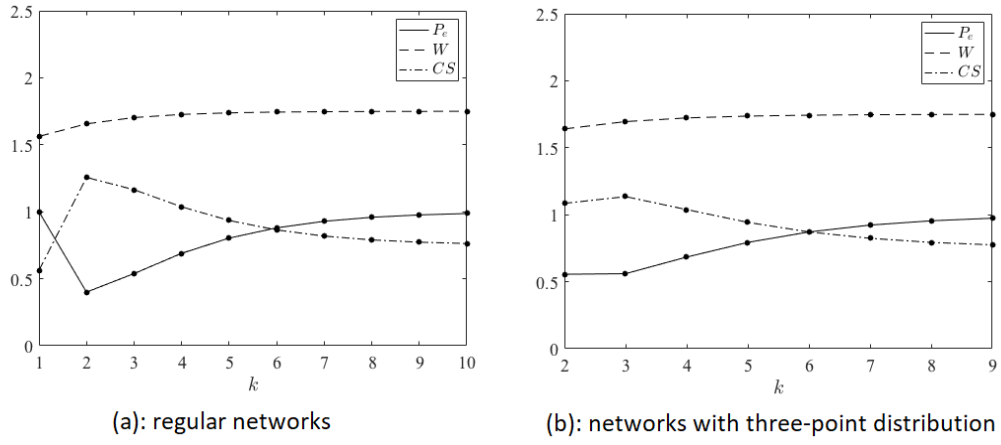


Figure 1: Comparative Statics

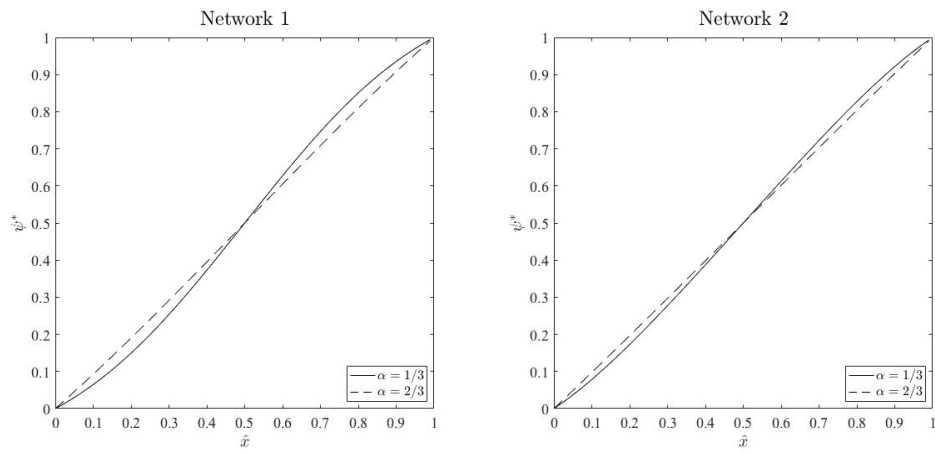


Figure 2: The $\psi(\hat{x})$ Curve as α Changes

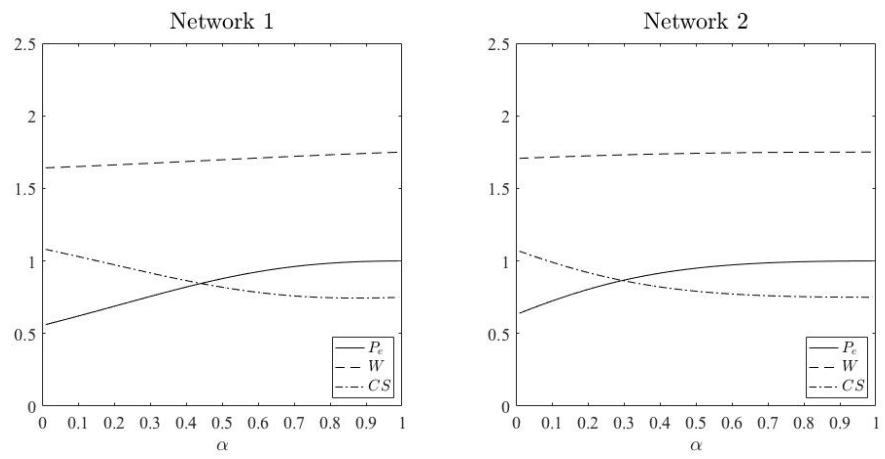


Figure 3: Homophily and Welfare with Symmetric Firms