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▶ To cite this version:

Bo Chen, Emilios Galariotis, Lijun Ma, Zijia Wang, Zhaobo Zhu. On disclosure of participation in innovation contests: a dominance result. Annals of Operations Research, 2023, 328 (2), pp.1615 - 1629. 10.1007/s10479-023-05416-0. hal-04185528

HAL Id: hal-04185528 https://audencia.hal.science/hal-04185528

Submitted on 26 Aug 2023 $\,$

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On Disclosure of Participation in Innovation Contests: A Dominance Result^{*}

Bo Chen[†] Emilios Galariotis[‡] Lijun Ma[§] Zijia Wang[¶] Zhaobo Zhu^{\parallel}

August 22, 2023

Abstract

This paper examines the effects of disclosing the actual number of participants in innovation contests with endogenous stochastic entry. We model innovation contests as a two-bidder all-pay auction with complete information, but in which each bidder has to incur a private cost to participate. The contest organizer observes solvers' participation decisions ex post and can commit ex ante to either fully disclosing or concealing the number of participating solvers. We characterize the equilibrium behavior of the solvers and compare the performances of the disclosure policies by four criteria. We find that full concealment dominates full disclosure in terms of expected total bid and expected winner's bid. Full concealment is dominated by full disclosure in terms of prize allocation efficiency and solvers' welfare. These findings are in sharp contrast to those under exogenous entry.

Keywords: auctions/bidding, contest, crowdsourcing, stochastic entry, disclosure.

^{*}We thank two anonymous referees and the editor for helpful suggestions. This work was supported by the National Natural Science Foundation of China [No. 72203148 (Chen); No. 71871145 (Ma)], the Department of Education of Guangdong Province [No. 2021WQNCX085], Shenzhen Science and Technology Program [No. 20220804114140001]. Wang acknowledges the support of the Digital Economy Platform, Major Innovation & Planning Interdisciplinary Platform for the "Double-First Class" Initiative. Zhu acknowledges the support of Shenzhen Humanities & Social Sciences Key Research Bases. The paper was partially done when Chen was serving Southern Methodist University, KFUPM, and Shenzhen MSU-BIT University, and he is grateful for all their support. Xilong Wu at Shenzhen MSU-BIT University provided excellent assistance to this work.

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Introduction

Companies and governments quite often sponsor or organize innovation contests. In these contests, solvers spend irreversible efforts in order to win some prizes.¹ Such contests have triggered broad interest in the optimal design of contest schemes and prize allocation rules to boost solvers' performances (e.g., Moldovanu and Sela, 2001; Abrache et al., 2007; Li and Kuo, 2013; Bonomo et al., 2017; Koçyiğit et al., 2018). The literature conventionally focuses on contests with a fixed set of solvers. However, potential solvers' availability is often subject to exogenous or endogenous randomness. In a government-organized R&D tournament, a participating firm may not be aware of the number of other firms who are also invited by the government to submit proposals. Firms can be absent from the competition for various nonstrategic reasons. For example, a research lab interested in an innovative project may undergo a sudden budget freeze imposed by higher authorities and be forced to quit the race. It can also be for some strategic reasons. For example, a research lab may have to exert effort to convince higher authorities to approve their budget proposal before they can proceed with their innovative project.

This paper studies whether an innovation contest organizer can boost solvers' performances by disclosing his exclusive information about solvers' participation decisions. We model innovation contests as a two-bidder all-pay auction with complete information, but in which each potential solver has to incur a private cost to participate. Only the organizer knows the actual number of participants, and the organizer has to announce at the very beginning whether he will fully disclose or conceal the number of participants and has to commit to his announcement throughout the game. The comparison across the disclosure policies focuses on four performance measures:

- Ex ante expected total bid (or aggregate expected effort) (e.g., Moldovanu and Sela, 2001): A benevolent government or firm might want to stimulate the market of research and thus cares about the aggregate effort of the contenders, in which case the losers' efforts are equally valued as the winner's effort.² Revenue maximizing contest design has been a focus of researchers in economics.
- Ex ante expected winner's bid: A company who organizes an innovation contest to procure some innovation (i.e., crowdsourcing contests) only values the winner's innovation (or proposal) and does not value those of the losers. Contest design on maximizing winner's bid has been the focus of researchers in operations management (e.g., Ales et al., 2017; Chawla et al., 2019; Fu and Wu, 2020; Körpeoğlu and Cho, 2018; Serena, 2017; Terwiesch and Xu, 2008).
- Prize allocation efficiency (or ex ante expected value realization of the prize): A

¹See Segev (2020) for a comprehensive survey on innovation contests.

 $^{^2\,{\}rm ``Revenue"}$ and ''bids'' in this paper correspond to non-contractible efforts but not to contractible monetary transfers.

government who organizes a contest to allocate a contract to a winner might care about the allocation efficiency.

• Ex ante expected payoff to the solvers (or social welfare): A social planner typically aims to maximize the social welfare.

We find that under endogenous stochastic participation, compared to fully disclosing the information on the actually number of participating solvers, full concealment leads to a higher expected total bid and a higher expected winner's bid but a lower allocation efficiency level and a lower aggregate expected payoff to the solvers. The key insight is that in the case in which solvers' entry decisions are endogenous, full concealment provides more incentive to the weak solver to participate and less incentive to the strong solver and thus can amplify the effect of leveling the playing field of the solvers, making the competition more intense.

Our study belongs to the literature that examines the effect of disclosing the actual number of participating contenders in contests. We extend the analysis of this issue into a setting in which solvers are asymmetric and their entry probabilities are endogenously determined, and we contribute novel performance ranking results to the literature.

Our study is most closely related to the work of Fu et al. (2016), who also investigate the effect of disclosing the actual number of participating contenders in a setting with asymmetric contenders. In their two-contender model, contenders differ in both their valuations of the prize and their exogenous probabilities of participation. Their key insight is that, under exogenous probabilities of participation, the performance rankings of the disclosure policies depend on the solvers' relative valuation and likelihood ratio of participation. More specifically, concealing the information on the actual number of participating contenders elicits a higher expected total bid than fully disclosing the information if and only if the value of the prize to the contender who has a higher probability of entry is sufficiently higher than that of the contender with a lower probability of entry. This is due to the trade-off between two effects of information disclosure — while information disclosure can eliminate information rent of the contenders, which helps the organizer to extract more efforts from the contenders, it would also unlevel the playing field, which potentially discourages the contenders from exerting efforts. Moreover, in certain scenarios, information disclosure could benefit both the organizer and the contenders at the same time.³ Our study goes beyond their work by extending the analysis to a setting with endogenous entry decisions and providing some new insights on the effects of disclosure policies in this case. In contrast to the findings in the case with exogenous entry, we find that information disclosure benefits the contenders and hurts the organizer in the case with endogenous entry.

Besides the work of Fu et al. (2016), the issue has also been scrutinized by many other

 $^{{}^{3}}$ Fu et al. (2016) only provides the revenue ranking. We complete the rankings in the other three performance measures in Proposition 4 in the Appendix.

studies. Lim and Matros (2009) and Fu et al. (2011) study information disclosure policies in Tullock contests in which each contender enters a contest with an exogenous probability. They relate optimal disclosure policies to the curvature of a characteristic function. Chen et al. (2017) go beyond these papers by introducing interdependent valuations of the prize with private and affiliated signals. They link optimal disclosure policies with the curvature of the cost function under both exogenous and endogenous probabilities of participation. Feng and Lu (2016) relate optimal disclosure policies to bidders' risk attitudes. Chen et al. (2020) go beyond them by considering disclosure policies in the situation in which bidders face a common bid cap. All these above mentioned studies focus on symmetric settings, in which equally competent contenders enter the contest with the same probability. In contrast to these studies, we investigate the cases in which contenders are asymmetric.

1 Model

Consider a two-stage contest with two potential risk-neutral solvers, 1 and 2, and one indivisible prize. Each solver k values the prize at $v_k < 1$. In the first stage, each solver decides whether to participate in the contest. In order to participate, solver k has to incur an entry cost of c_k , which is solver k's private information and is independently drawn from the uniform distribution over [0, 1]. Each solver's entry is unobservable to her rival but observed by the contest organizer. The organizer has to announce publicly and commit to his disclosure policy — either to **fully conceal (Policy** C) or **fully disclose** (**Policy** D) the information on the actual number of participants.

In the second stage, each participating solver k exerts an effort (bid) b_k . Bids are submitted simultaneously and independently of each other. The solver with the highest bid wins the prize, but all participating solvers pay their bids. Ties are resolved by random allocation with equal probabilities. When there is a subset M of participating solvers, solver i bids b_i and obtains an interim expected utility (conditional on her participation)

$$U_{i} = \begin{cases} v_{i} - b_{i} & \text{if } b_{i} > \max_{j \in M \setminus \{i\}} b_{j} \\ -b_{i} & \text{if } b_{i} < \max_{j \in M \setminus \{i\}} b_{j} \\ \frac{1}{\#\{k \in M: b_{k} = b_{i}\}} \cdot v_{i} - b_{i} & \text{if } b_{i} = \max_{j \in M \setminus \{i\}} b_{j}. \end{cases}$$

In detail, the timing of the game is as the following:

- 1. The organizer publicly announces his disclosure policy before the contest starts and commits to the announced policy throughout the game.
- 2. Each solver decides whether to participate in the contest.
- 3. The organizer learns the number of participants and implements his announced disclosure policy.⁴

⁴It is beyond the scope of the study to provide a thorough analysis of the issue of commitment.

- 4. Solvers submit their bids privately.
- 5. The one with the highest bid wins the prize, and ties are resolved by fair lotteries.

Before proceeding to endogenous participation, we present below the equilibrium behavior under exogenous probabilities of participation, which will be used later.

Lemma 1 (Equilibrium under Exogenous Probabilities of Participation (Fu et al., 2016)). Consider the case in which the organizer implements policy C. Suppose that it is common knowledge that solver 1 believes that bidder 2 participates with probability p_2 and that solver 2 believes that solver 1 participates with probability p_1 . There is a unique Nash equilibrium ($\mu_1^*(p_1, p_2), \mu_2^*(p_1, p_2)$) of the second stage (the bidding stage). If $p_j v_i \leq p_i v_j$, where $i \neq j$ and $i, j \in \{1, 2\}$, then solver i, if she has entered, bids 0 with probability $\frac{p_i v_j - p_j v_i}{p_i v_j}$ and randomizes uniformly on $[0, p_j v_i]$ with probability $\frac{p_j v_i}{p_i v_j}$; solver j, if she has entered, randomizes uniformly on $[0, p_j v_i]$.

2 Results

Now, let us analyze our model of all-pay auction with endogenous entry. First, consider the subgame in which the organizer commits to policy C. In this case, the organizer conceals the actual number of solvers before the participating solvers make their bids.

The second stage strategies of the participating solvers are described as in Lemma 1. The first stage strategies of the solvers must be cutoff strategies, i.e., each solver k participates if and only if her cost is below a certain threshold value \hat{c}_k^C . Since each solver's participation cost is drawn from the uniform distribution over [0, 1], the value \hat{c}_k^C also corresponds to solver k's probability of participation. To find an equilibrium, we only need to pin down the equilibrium cutoffs \hat{c}_1^C and \hat{c}_2^C .

Proposition 1 (Full Concealment). Suppose $v_1 \ge v_2$. Consider the subgame that follows policy C. There exists a unique Bayesian Nash equilibrium. Let $([\hat{c}_1^C, \mu_1^C], [\hat{c}_2^C, \mu_2^C])$ be a Nash equilibrium.

1.
$$(\mu_1^C, \mu_2^C) = (\mu_1^*(p_1^C, p_2^C), \mu_2^*(p_1^C, p_2^C)).$$

2. $(p_1^C, p_2^C) = (\hat{c}_1^C, \hat{c}_2^C) = (v_1 - \frac{v_1 v_2}{1 + v_1}, \frac{v_2}{1 + v_1}).$

The key observations in the above proposition are that: (1) solver 1, who has a higher valuation, has a higher probability of participation; (2) solver 2, who has a lower valuation, bids more aggressively upon entering the contest (i.e., because $p_2^C v_1 \leq p_1^C v_2$, solver 2 does not bid 0, whereas solver 1 bids 0 with a positive probability).

Let us then consider the subgame in which the organizer commits to policy D. In this case, the organizer discloses the actual number of solvers before the participating solvers make their bids. Here, the second stage is a standard all-pay auction with complete information. Again, solvers' first stage strategies must be cutoff strategies. To find an equilibrium, we only need to pin down the equilibrium cutoffs \hat{c}_1^D and \hat{c}_2^D .

Proposition 2 (Full Disclosure). Suppose $v_1 \ge v_2$. Consider the subgame that follows policy D. Take the equilibrium of the second stage as given in Lemma 1. There is a unique Bayesian Nash equilibrium in the first stage $(\hat{c}_1^D, \hat{c}_2^D) = (\frac{v_1 - v_2^2}{1 - v_2^2}, \frac{(1 - v_1)v_2}{1 - v_2^2}).$

Under full disclosure, it is also solver 1, the stronger solver, that has a higher probability of participation.

The following main result compares the two disclosure policies in all the four performance measures as described in the introduction.

Theorem 1 (Performance Rankings under Endogenous Entry). Consider the contest with endogenous entry.

- (i) The ex ante expected bid of each solver, as well as the ex ante expected total bid, is higher (and strictly higher if $v_1 \neq v_2$) under full concealment than under full disclosure.
- (ii) The ex ante expected winner's bid is strictly higher under full concealment than under full disclosure.
- (iii) The allocation efficiency is lower (and strictly lower if $v_1 \neq v_2$) under full concealment than under full disclosure.
- (iv) The ex ante expected total payoff of the solvers is lower (and strictly lower if $v_1 \neq v_2$) under full concealment than under full disclosure.

In contrast to the finding for the case with exogenous entry, where the rankings of the disclosure policies crucially depend on the solvers' relative valuation of the prize and information disclosure could benefit both the organizer and the solvers, Theorem 1 shows that once solvers' entry decisions are endogenously determined by themselves full disclosure unambiguously hurts the organizer and benefits the solvers.⁵ Compared to fully concealing the information on participation, full disclosure has two effects: (1) it could further unlevel the playing field of the two solvers and thus discourages the bidding of both solvers; (2) it reduces the overall participation of the solvers. Though full disclosure increases the allocation efficiency, the organizer could not benefit from it, as his ability to extract surplus from the solvers is significantly lowered by this policy.

To give the intuition, suppose $v_1 > v_2$. Solver 1 is stronger than solver 2, and, naturally, solver 1 has more incentive to participate than solver 2, under either disclosure policy. The more likely solver 1 is to participate (and the more unlikely solver 2 is to participate), the more unbalanced the contest is, which would alleviate the competition between the two solvers. Compared to full concealment, full disclosure provides more incentive to the strong solver (solver 1) and less incentive to the weak solver (solver 2) to participate (i.e., $\hat{c}_1^D > \hat{c}_1^C$ and $\hat{c}_2^D < \hat{c}_2^C$), which further sharpens the advantage of the strong solver over the weak one, making the playing field even more unbalanced.

 $^{^5\}mathrm{See}$ Proposition 4 in the Appendix for details on the case with exogenous probabilities of participation.

Moreover, the discouragement effect of full disclosure on solver 2 is so strong that it elicits a lower overall participation rate than under full concealment (i.e., $1 - (1 - \hat{c}_1^D)(1 - \hat{c}_2^D) < 1 - (1 - \hat{c}_1^C)(1 - \hat{c}_2^C)$). First, the decrease in the overall participation rate seems to decrease the overall allocation efficiency. However, this is mitigated by the gain from a higher winning probability of the strong solver, which is so significant that it increases the overall allocation efficiency. Second, the effects of unbalancing-the-playing-field and decrease-in-participation together alleviate the competition and elicit less efforts from the solvers, which increases bidders' total payoff but decreases both the ex ante expected total effort and ex ante expected winner's bid.

In the following, we further examine the effects of information disclosure on solvers' welfare. The result is summarized in the proposition below.

Proposition 3. Suppose $v_1 > v_2$. Solver 1's (solver 2's) ex ante expected payoff and probability of winning are strictly lower (higher) under full concealment than under full disclosure.

Proposition 3 compares each solver's welfare, both in terms of the solvers' expected payoffs and their winning probabilities, across the two disclosure policies. Here, disclosing "more" information about solvers' participation decisions benefits the strong solver and harms the weak one. The intuition is that information disclosure eliminates the weak solver's information rent (at the bidding stage) and sharpens the strong solver's valuation advantage, reducing the weak solver's incentive to participate and increasing that of the strong solver.

There are two implications of Proposition 3 that are worth mentioning explicitly. First, the interest of the organizer is aligned with that of the weak solver who is at a disadvantage, as both of them would like to attenuate the advantage of the strong solver, but conflicts with that of the strong solver who is at an advantage. Second, in our all-pay auction, a solver wins the prize if and only if her (realized) bid is higher than that of her opponent. As a result, the change in a solver's expected payoff is perfectly aligned with the change in the solver's winning probability in equilibrium across the information structures.

Proposition 3 resonates the insights of Milgrom and Weber (1982) and Wärneryd (2003) on disclosure of players' private signals in a common-value first price auction and a common-value contest, respectively, with one-sided private information on the value of a prize. In our setting, the solver with a higher valuation has a valuation advantage, and concealing the information about participation (relative to disclosing it) would attenuate the strong solver's advantage and elicit more effort from the solvers. Unlike our setting, in the settings of Milgrom and Weber (1982) and Wärneryd (2003), the player who has private information about the value of the prize is at an informational advantage (and the one who has no private information at a disadvantage), and fully

disclosing the private information would attenuate this player's advantage and intensify the competition. However, similar to our finding, both papers demonstrate that both the organizer and the uninformed player with informational disadvantage prefer full revealing in order to attenuate the informational advantage of the informed player, whereas the informed player has exactly the opposite preferences.

Finally, on solvers' winning probabilities under the two information structures, Proposition 3 is parallel to the finding of Milgrom and Weber (1982), which shows that a payoff gain (loss) of a player from information disclosure is associated with a gain (loss) in probability of winning in their first-price auction.⁶

3 Conclusion

In this note, we examined the effects of disclosing the actual number of solvers in an innovation contest with endogenous entry of two asymmetric solvers. It is shown that the organizer prefers fully concealing the information about the number of participating solvers to fully disclosing the information, whereas solvers prefer full disclosure to full concealment.

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⁶This result differs drastically from that in Wärneryd (2003). To be specific, Wärneryd (2003) shows that in a contest with common value and one-sided private information, a payoff gain (loss) of a player from information disclosure is associated with a loss (gain) in the probability of winning. This is actually due to the noisy winning criterion of a contest (in contrast to that of an all-pay auction). In a Tullock lottery contest, for example, a player with a lower bid still has some chance to win the contest, and maximizing expected payoffs is hence less associated with the probability of winning.

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Appendix

Part A: Exogenous Entry

First, following Lemma 1, we write out the performances of the equilibrium under exogenous entry below.

Corollary 1 (Performances under Exogenous Probabilities of Participation). Consider the case in which the organizer implements policy C. Suppose that it is common knowledge that solver 1 believes that bidder 2 participates with probability p_2 and that solver 2 believes that solver 1 participates with probability p_1 . Suppose $p_jv_i \leq p_iv_j$. In the unique Nash equilibrium, the ex ante expected total bid is

$$R^{C}(p_{j}, p_{i}) = \frac{p_{j}^{2}v_{i}(v_{i} + v_{j})}{2v_{j}};$$

the ex ante expected winner's bid is

$$W^{C}(p_{j}, p_{i}) = \left(\frac{p_{j}}{2} + \frac{p_{j}v_{i}}{2v_{j}} - \frac{p_{j}^{2}v_{i}}{3v_{j}}\right)p_{j}v_{i};$$

the expected value realization of the prize is⁷

$$V^{C}(p_{j}, p_{i}) = p_{i}(1 - p_{j})v_{i} + (1 - p_{i})p_{j}v_{j} + p_{i}p_{j}\left(\frac{p_{j}v_{i}}{p_{i}v_{j}} \cdot \frac{v_{i} + v_{j}}{2} + \frac{p_{i}v_{j} - p_{j}v_{i}}{p_{i}v_{j}} \cdot v_{j}\right);$$

the expected payoff of solver k, k = 1, 2, conditional on that she has entered is

$$U_k^C(p_j, p_i) = v_k - p_j v_i.$$

Second, we complete the rankings in all performance measures.

Proposition 4 (Rankings under Exogenous Probabilities of Participation). Suppose solver 1 and solver 2 participate with probabilities p_1 and p_2 , respectively. Suppose $\frac{p_i}{p_i} \ge 1$:

- (i) The ex ante expected revenue under full concealment is strictly higher than (the same as) under full disclosure if and only if $\frac{v_i}{v_j} > (=) \sqrt{\frac{p_i}{p_j}}$.
- (ii) There is a unique $h \in [0, \frac{p_i}{p_j})$, with h = 0 if and only if $p_i = p_j$, such that the exact ante expected winner's bid under full concealment is strictly higher than (the same as) under full disclosure if and only if $\frac{v_i}{v_j} > (=)h$.
- (iii) If $\frac{p_i}{p_j} > 1$, the allocation efficiency under full disclosure is strictly higher than (the same as) under full disclosure if and only if $\frac{v_i}{v_j} > (=)1$. If $\frac{p_i}{p_j} = 1$, the ex ante expected winner's bid remains the same across the two disclosure policies.

⁷The ex ante expected value realization of the prize is $\sum_{k=1}^{2} Pr\{solver \ k \ wins\}v_k$.

(iv) If $\frac{p_i}{p_j} > 1$, the ex ante expected total payoff of the bidders under full disclosure is strictly higher than (the same as) under full concealment if and only if $\frac{v_i}{v_j} > (=)$ $\frac{2p_i}{p_i+p_j}$. If $\frac{p_i}{p_j} = 1$, the ex ante expected winner's bid remains the same across the two disclosure policies.

Proof of Proposition 4.

Note that $p_i \ge p_j$ (and $p_i p_j \in (0, 1)$).

[1] Expected Revenue Ranking under Exogenous Entry: See Fu et al. (2016).

[2] Expected Winner's Bid Ranking under Exogenous Entry: Let $s := \frac{v_i}{v_j}$ and $G(s) := W^C(p_i, p_j) - W^D(p_i, p_j)$. Then,

$$G(s) = \begin{cases} \left(\frac{p_i}{2} + \frac{p_i v_j}{2v_i} - \frac{p_i^2 v_j}{3v_i}\right) p_i v_j - \left(\frac{v_j}{2} + \frac{v_j^2}{6v_i}\right) = \frac{p_i v_j [(p_i - p_j)(3v_i + v_j) + 2p_i(1 - p_i)v_j]}{6v_i} & \text{for } s \ge \frac{p_i}{p_j} \\ \left(\frac{p_j}{2} + \frac{p_j v_i}{2v_j} - \frac{p_j^2 v_i}{3v_j}\right) p_j v_i - \left(\frac{v_j}{2} + \frac{v_j^2}{6v_i}\right) = \frac{p_i v_j^2 [(3p_j - 2p_j^2)s^3 + 3p_js^2 - 3p_is - p_i]}{6v_i} & \text{for } s \in [1, \frac{p_i}{p_j}) \\ \left(\frac{p_j}{2} + \frac{p_j v_i}{2v_j} - \frac{p_j^2 v_i}{3v_j}\right) p_j v_i - \left(\frac{v_i}{2} + \frac{v_i^2}{6v_j}\right) = p_j v_i \left[3(p_j - p_i) + (3p_j - 2p_j^2 - p_i)s\right] & \text{for } s \in [0, 1). \end{cases}$$

 $G(\cdot)$ is continuous, with G(0) < 0 and G(s) > 0 for $s \ge \frac{p_i}{p_j}$. It can be verified that G(s) satisfies the single crossing property on [0,1] and $[1,\frac{p_i}{p_j})$. Then, if $G(1) \ge 0$, there is a $x \in (0,1]$ such that G(s) > 0 for $s \in (x,\frac{p_i}{p_j}]$, < 0 for $s \in [0,k)$, and = 0 for s = k. Similarly, if G(1) < 0, there is a $x \in (1,\frac{p_i}{p_j})$ such that G(s) > 0 for $s \in (x,\frac{p_i}{p_j}]$, < 0 for $s \in [0,x)$, and = 0 for s = x.

[3] Allocation Efficiency Ranking under Exogenous Entry: First, suppose $\frac{v_i}{v_j} \leq 1 \leq \frac{p_i}{p_j}$:

$$\begin{aligned} V^{C}(p_{i},p_{j}) - V^{D}(p_{i},p_{j}) &= \left[p_{i}(1-p_{j})v_{i} + (1-p_{i})p_{j}v_{j} + p_{i}p_{j}\left(\frac{p_{j}v_{i}}{p_{i}v_{j}} \cdot \frac{v_{i} + v_{j}}{2} + \frac{p_{i}v_{j} - p_{j}v_{i}}{p_{i}v_{j}} \cdot v_{j} \right) \right] \\ &- \left[p_{i}(1-p_{j})v_{i} + (1-p_{i})p_{j}v_{j} + p_{i}p_{j}\left(\frac{v_{i}}{v_{j}} \cdot \frac{v_{i} + v_{j}}{2} + \frac{v_{j} - v_{i}}{v_{j}} \cdot v_{j} \right) \right] \\ &= \frac{p_{j}v_{i}(v_{j} - v_{i})(p_{i} - p_{j})}{2v_{j}} \\ \geq 0 \quad (= 0 \text{ iff } v_{i} = v_{j} \text{ or } p_{i} = p_{j}). \end{aligned}$$

Second, suppose $1 < \frac{v_i}{v_j} \le \frac{p_i}{p_j}$:

$$V^{C}(p_{i}, p_{j}) - V^{D}(p_{i}, p_{j}) = \left[p_{i}(1 - p_{j})v_{i} + (1 - p_{i})p_{j}v_{j} + p_{i}p_{j}\left(\frac{p_{j}v_{i}}{p_{i}v_{j}} \cdot \frac{v_{i} + v_{j}}{2} + \frac{p_{i}v_{j} - p_{j}v_{i}}{p_{i}v_{j}} \cdot v_{j}\right)\right]$$

$$\begin{split} & -\left[p_i(1-p_j)v_i + (1-p_i)p_jv_j + p_ip_j\left(\frac{v_j}{v_i} \cdot \frac{v_i + v_j}{2} + \frac{v_i - v_j}{v_i} \cdot v_i\right)\right] \\ = & \frac{p_j(v_i - v_j)(p_jv_i^2 + p_iv_j^2 - 2p_iv_iv_j)}{2v_iv_j} \\ < & 0, \end{split}$$

$$\begin{aligned} \text{as } \frac{p_i - \sqrt{p_i^2 - p_i p_j}}{p_j} < 1 \text{ and } \frac{p_i + \sqrt{p_i^2 - p_i p_j}}{p_j} > \frac{p_i}{p_j}. \text{ Third, suppose } 1 \le \frac{p_i}{p_j} < \frac{v_i}{v_j}: \\ V^C(p_i, p_j) - V^D(p_i, p_j) = \left[p_i(1 - p_j)v_i + (1 - p_i)p_jv_j + p_ip_j \left(\frac{p_i v_j}{p_j v_i} \cdot \frac{v_i + v_j}{2} + \frac{p_j v_i - p_i v_j}{p_j v_i} \cdot v_i \right) \right] \\ &- \left[p_i(1 - p_j)v_i + (1 - p_i)p_jv_j + p_ip_j \left(\frac{v_j}{v_i} \cdot \frac{v_i + v_j}{2} + \frac{v_i - v_j}{v_i} \cdot v_i \right) \right] \\ &= - \frac{p_i v_j (v_i - v_j)(p_i - p_j)}{2v_i} \\ \le 0 \quad (= 0 \text{ if } p_i = p_j). \end{aligned}$$

[4] Welfare Ranking under Exogenous Entry: First, suppose $\frac{v_i}{v_j} \le 1 \le \frac{p_i}{p_j}$:

$$U^{C}(p_{i}, p_{j}) - U^{D}(p_{i}, p_{j}) = [p_{i}(v_{i} - p_{j}v_{i}) + p_{j}(v_{j} - p_{j}v_{i})] - [p_{i}(1 - p_{j})v_{i} + p_{j}(1 - p_{i})v_{j} + p_{i}p_{j}(v_{i} - v_{j})]$$
$$= p_{j}(p_{i} - p_{j})v_{i}$$
$$\geq 0 \quad (= 0 \text{ if } p_{i} = p_{j}).$$

Second, suppose $1 < \frac{v_i}{v_j} \le \frac{p_i}{p_j}$:

$$U^{C}(p_{i}, p_{j}) - U^{D}(p_{i}, p_{j}) = [p_{i}(v_{i} - p_{j}v_{i}) + p_{j}(v_{j} - p_{j}v_{i})] - [p_{i}(1 - p_{j})v_{i} + p_{j}(1 - p_{i})v_{j} + p_{i}p_{j}(v_{i} - v_{j})]$$
$$= p_{j}(p_{i} + p_{j})v_{j}\left(\frac{2p_{i}}{p_{i} + p_{j}} - \frac{v_{i}}{v_{j}}\right).$$

Note that $\frac{2p_i}{p_i+p_j} \leq \frac{p_i}{p_j}$ for $p_i \geq p_j$. In this case $U^C - U^D$ is less than 0 if $\frac{v_i}{v_j} > \frac{2p_i}{p_i+p_j}$, larger than 0 if $\frac{v_i}{v_j} < \frac{2p_i}{p_i+p_j}$, and equals to 0 if $\frac{v_i}{v_j} = \frac{2p_i}{p_i+p_j}$. Third, suppose $1 \leq \frac{p_i}{p_j} < \frac{v_i}{v_j}$:

$$U^{C}(p_{i}, p_{j}) - U^{D}(p_{i}, p_{j}) = [p_{i}(v_{i} - p_{i}v_{j}) + p_{j}(v_{j} - p_{i}v_{j})] - [p_{i}(1 - p_{j})v_{i} + p_{j}(1 - p_{i})v_{j} + p_{i}p_{j}(v_{i} - v_{j})]$$

$$= -p_{i}(p_{i} - p_{j})v_{j}$$

$$\leq 0 \quad (= 0 \text{ if } p_{i} = p_{j}).$$

The above result shows that a gain in the organizer's benefit from information disclosure (or information concealment) is not necessarily associated with a gain in allocation efficiency or a loss in the solvers' total payoff. Note that $h < \frac{p_i}{p_j} < \frac{2p_i}{p_i + p_j} < \sqrt{\frac{p_i}{p_j}}$ as long as $\frac{v_i}{v_j} \neq 1$. If the solvers' valuations are neither too similar nor too different (e.g., $\frac{v_i}{v_j} \in (\frac{2p_i}{p_i + p_j}, \sqrt{\frac{p_i}{p_j}})$ in case that the organizer is a benevolent sponsor), full disclosure is actually a win-win policy for the organizer and the solvers, under which both the organizer and the solvers are better off, and it also leads to a higher level of allocation efficiency. However, if $\frac{v_i}{v_j} < 1$, then at the time when full disclosure increases the benefit of the organizer, it reduces the allocation efficiency.

Part B: Endogeous Entry

Proof of Proposition 1.

First, each solver makes her participation decision and bidding decision sequentially. Solvers' second stage bidding behavior follows from Lemma 1. We only need to pin down the equilibrium probabilities of participation (or the cutoff participation costs).

Suppose $p_1^C v_2 \ge p_2^C v_1$. Given that solver 1 participates with probability p_1^C and believes that solver 2 participates with probability p_2^C , solver 2's expected payoff will be

$$(v_2 - p_2^C v_1) - c_2$$

if she participates and 0 otherwise. She strictly prefers (not) taking part in the contest if $(v_2-p_2^Cv_1)-c_2 > (<)0$ and is indifferent between participating and not if $(v_2-p_2^Cv_1)-c_2 = 0$. Thus, we can derive the cutoff participation cost of solver 2 can be derived as follows:

$$(v_2 - p_2^C v_1) - \hat{c}_2^C = 0 \Rightarrow (v_2 - \hat{c}_2^C v_1) - \hat{c}_2 = 0 \Rightarrow \hat{c}_2^C = \frac{v_2}{1 + v_1}$$

On the other hand, given that solver 2 participates with probability p_2^C and believes that solver 1 participates with probability p_1^C , solver 1's expected payoff will be

$$(v_1 - p_2^C v_1) - c_1$$

if she participates and 0 otherwise. She strictly prefers (not) taking part in the contest if $(v_1-p_2^Cv_1)-c_1 > (<)0$ and is indifferent between participating and not if $(v_1-p_2^Cv_1)-c_1 = 0$. Thus, we can derive the cutoff participation cost of solver 1 can be derived as follows:

$$(v_1 - p_2^C v_1) - \hat{c}_1^C = 0 \Rightarrow (v_1 - \hat{c}_2 v_1) - \hat{c}_1^C = 0 \Rightarrow \hat{c}_1^C = v_1 - \frac{v_1 v_2}{1 + v_1}.$$

Last, we show that $v_1 \ge v_2$.

$$p_1^C v_2 \ge p_2^C v_1 \Rightarrow \left[v_1 - \frac{v_1 v_2}{1 + v_1} \right] v_2 \ge \frac{v_2}{1 + v_1} \cdot v_1$$
$$\Rightarrow v_1 + v_1^2 - v_1 v_2 \ge v_1$$

$$\Rightarrow v_1 \ge v_2 \ (p_1^C v_2 = p_2^C v_1 \text{ iff } v_1 = v_2).$$

Proof of Proposition 2.

Suppose $v_1 \ge v_2$. Given that solver 2 participates with a probability of p_2^D , solver 1's expected payoff is

$$\left[(1 - p_2^D) \cdot v_1 + p_2^D \cdot (v_1 - v_2) \right] - c_1$$

if she participates and 0 otherwise. She strictly prefers (not) taking part in the contest if $[(1-p_2^D) \cdot v_1 + p_2^D \cdot (v_1 - v_2)] - c_1 > (<)0$. She is indifferent between participating and not if $[(1-p_2^D) \cdot v_1 + p_2^D \cdot (v_1 - v_2)] - c_1 = 0$.

Given that solver 1 participates with a probability of p_1^D , solver 2's expected payoff is

$$(1-p_1^D) \cdot v_2 - c_2$$

if she participates and 0 otherwise. She strictly prefers (not) taking part in the contest if $(1-p_1^D)\cdot v_2-c_2 > (<)0$. She is indifferent between participating and not if $(1-p_1^D)\cdot v_2-c_2 = 0$.

Thus, we can derive the cutoff participation costs of solver 1 and solver 2 can be derived by solving the two equations and two unknowns below:

$$(v_1 - \hat{c}_2^D v_2) - \hat{c}_1^D = 0;$$

 $(1 - \hat{c}_1^D) \cdot v_2 - \hat{c}_2^D = 0.$

Hence, the cutoffs are

$$\hat{c}_1^D = \frac{v_1 - v_2^2}{1 - v_2^2}; \\ \hat{c}_2^D = \frac{(1 - v_1)v_2}{1 - v_2^2}.$$

Thus, we can derive the cutoff participation cost of solver 2' can be derived as follows:

$$(1 - p_1^D) \cdot v_2 - \hat{c}_2^D = 0 \Rightarrow (1 - \frac{v_1}{1 + v_2}) \cdot v_2 - \hat{c}_2^D = 0 \Rightarrow \hat{c}_2^D = v_2 - \frac{v_1 v_2}{1 + v_2}.$$

Following from Propositions 1 and 2 and Lemma 1, we state the performances of the two disclosure policies under endogenous entry below.

Corollary 2. Suppose $v_1 \ge v_2$.

i. In the equilibrium under full concealment as stated in Proposition 1: The ex ante expected total bid of the equilibrium is

$$R^C = \frac{v_1 v_2 (v_1 + v_2)}{2(1 + v_1)^2}.$$

The ex ante expected winner's bid is

$$W^{C} = \frac{v_{1}v_{2}(v_{1}v_{2} + 3v_{2} + 3v_{1}^{2} + 3v_{1})}{6(1+v_{1})^{3}}.$$

The expected value realization of the prize is

$$V^{C} = \hat{c}_{1}^{C} (1 - \hat{c}_{2}^{C}) v_{1} + (1 - \hat{c}_{1}^{C}) \hat{c}_{2}^{C} v_{2} + \hat{c}_{1}^{C} \hat{c}_{2}^{C} \left(\frac{\hat{c}_{2}^{C} v_{1}}{\hat{c}_{1}^{C} v_{2}} \cdot \frac{v_{1} + v_{2}}{2} + \frac{\hat{c}_{1}^{C} v_{2} - \hat{c}_{2}^{C} v_{1}}{\hat{c}_{1}^{C} v_{2}} \cdot v_{2} \right)$$

The ex ante expected total payoff of solver k is⁸

$$U_k^C = \frac{\left(\hat{c}_k^C\right)^2}{2}.$$

ii. In the equilibrium under full disclosure as stated in Proposition 2: The ex ante expected total bid is

$$R^{D} = \frac{(1-v_1)(v_1-v_2^2)(v_1+v_2)v_2^2}{2(1-v_2^2)^2v_1}.$$

The ex ante expected winner's bid is

$$W^{D} = \frac{(1-v_{1})(v_{1}-v_{2}^{2})(3v_{1}+v_{2})v_{2}^{2}}{6(1-v_{2}^{2})^{2}v_{1}}.$$

The expected value realization of the prize is

$$V^{D} = \hat{c}_{1}^{D}(1 - \hat{c}_{2}^{D})v_{1} + (1 - \hat{c}_{1}^{D})\hat{c}_{2}^{D}v_{2} + \hat{c}_{1}^{D}\hat{c}_{2}^{D}\left(\frac{v_{2}}{v_{1}} \cdot \frac{v_{1} + v_{2}}{2} + \frac{v_{1} - v_{2}}{v_{1}} \cdot v_{1}\right).$$

The ex ante expected total payoff of solver k is

$$U_k^D = \frac{\left(\hat{c}_k^D\right)^2}{2}.$$

Proof of Theorem 1.

⁸Under full concealment, solver k's ex ante expected payoff is $p_k^C \cdot U_k^C(p_j^C, p_i^C) - p_i^C \frac{\hat{c}_i^C}{2} = \frac{(\hat{c}_i^C)^2}{2}$, as $p_i^C = U_i(p_i^C, p_j^C) = \hat{c}_i^C$. This is similar to the case under full disclosure.

[i] Revenue Ranking: Suppose $v_1 \ge v_2$. Solver 1's ex ante expected bid under full concealment minuses that under full disclosure is

$$\hat{c}_{1}^{C} \cdot \frac{\hat{c}_{2}^{C}v_{1}}{\hat{c}_{1}^{C}v_{2}} \cdot \frac{\hat{c}_{2}^{C}v_{1}}{2} - \hat{c}_{1}^{D}\hat{c}_{2}^{D} \cdot \frac{v_{2}}{2} = \frac{v_{2}(v_{1}-v_{2})\left[v_{2}^{2}+v_{1}(1+v_{1}v_{2}-v_{2}^{2}-v_{2})+v_{1}^{2}v_{2}(1+v_{2}-v_{2}^{2})\right]}{2(v_{1}+1)^{2}(1-v_{2})^{2}(v_{2}+1)^{2}} > 0$$

Solver 2's ex ante expected bid under full concealment minuses that under full disclosure is

$$\hat{c}_2^C \cdot \frac{\hat{c}_2^C v_1}{2} - \hat{c}_1^D \hat{c}_2^D \cdot \frac{v_2}{v_1} \cdot \frac{v_2}{2} = \frac{v_2}{v_1} \left[\hat{c}_1^C \cdot \frac{\hat{c}_2^C v_1}{\hat{c}_1^C v_2} \cdot \frac{\hat{c}_2^C v_1}{2} - \hat{c}_1^D \hat{c}_2^D \cdot \frac{v_2}{2} \right] > 0.$$

[ii] Winner's Bid Ranking: Let

$$S(x) := (v_1^3 + 3v_1^2)x^5 + (2v_1^4 + v_1^3 + 2v_1 + 1)x^4 - (3v_1^5 + 6v_1^4 + 2v_1^3 - 3v_1)x^3 + (v_1^5 - 4v_1^4 - 6v_1^3 - 2v_1^2 - v_1)x^2 + (3v_1^6 + 6v_1^5 - 5v_1^3)x + 3v_1^4 + 3v_1^3.$$

In particular, $S(0) = 3(1+v_1)v_1^3 > 0$ and $S(v_1) = 2(1-v_1^2)^2v_1^3 > 0$. It can be verified that S(x) satisfies the single crossing property on $[0, v_1]$. More specifically, S(x) = 0 implies S'(x) < 0. Hence, S(x) > 0 for all $x \in [0, v_1]$.

Next, the difference in expected winner's bid across the disclosure policies is:

$$W^{C} - W^{D} = \frac{v_{2}}{6(1+v_{1})^{3}(1-v_{2}^{2})^{2}v_{1}} \cdot S(v_{2}) > 0.$$

[iii] Allocation Efficiency Ranking: First, let

$$S(x) := (2v_1^3 + v_1^2 + 2v_1)x^4 - (2v_1^4 + 3v_1^3 - v_1^2 - v_1 - 1)x^3 - 4(v_1^3 + v_1^2 + v_1)x^2 + (5v_1^4 + 3v_1^3 - 5v_1^2 - v_1)x + 4v_1^3 + 3v_1^2$$

In particular, $S(0) = 4v_1^3 + 3v_1^2 > 0$ and $S(v_1) = 2(1 - v_1)^3(1 + v_1)v_1^2 > 0$. It can be verified that S(x) satisfies the single crossing property on $[0, v_1]$. More specifically, S(x) = 0 implies S'(x) < 0. Hence, S(x) > 0 for all $x \in [0, v_1]$.

Next, the difference in expected value realization of the prize across the disclosure policies is:

$$\begin{split} V^{C} - V^{D} &= \left[p_{i}^{C} (1 - p_{j}^{C}) v_{1} + p_{j}^{C} (1 - p_{i}^{C}) v_{2} + p_{i}^{C} p_{j}^{C} \left(\frac{p_{i}^{C} v_{2} - p_{j}^{C} v_{1}}{p_{i}^{C} v_{2}} \cdot v_{2} + \frac{p_{j}^{C} v_{1}}{p_{i}^{C} v_{2}} \cdot \frac{v_{1}}{2} + \frac{p_{j}^{C} v_{1}}{p_{i}^{C} v_{2}} \cdot \frac{v_{1}}{2} \right) \right] \\ &- \left[p_{i}^{D} (1 - p_{j}^{D}) v_{1} + p_{j}^{D} (1 - p_{i}^{D}) v_{2} + p_{i}^{D} p_{j}^{D} \left(\frac{v_{1} - v_{2}}{v_{1}} \cdot v_{1} + \frac{v_{2}}{v_{1}} \cdot \frac{v_{1}}{2} + \frac{v_{2}}{v_{1}} \cdot \frac{v_{2}}{2} \right) \right] \\ &= \frac{-(v_{1} - v_{2}) S(v_{2}) v_{2}}{2(1 + v_{1})^{2} (1 - v_{2}^{2})^{2} v_{1}} \end{split}$$

$$\leq 0 \ (= 0 \ \text{iff} \ v_1 = v_2).$$

[iv] Welfare Ranking: First,

$$(\hat{c}_1^D - \hat{c}_1^C), (\hat{c}_2^C - \hat{c}_2^D) \ge 0 \quad (= 0 \text{ iff } v_1 = v_2);$$

$$(\hat{c}_2^C - \hat{c}_2^D) - (\hat{c}_1^D - \hat{c}_1^C) = -\frac{(1 - v_1)(v_1 - v_2)v_2}{(1 + v_1)(1 + v_2)} \le 0 \quad (= 0 \text{ iff } v_1 = v_2);$$

$$(\hat{c}_2^C + \hat{c}_2^D) - (\hat{c}_1^D + \hat{c}_1^C) = -\frac{(2 - v_2)(v_1 - v_2)}{1 - v_2} \le 0 \quad (= 0 \text{ iff } v_1 = v_2),$$

Then, the difference in ex ante expected total payoff across the disclosure policies is:

$$(U_1^C + U_2^D) - (U_1^D + U_2^D) = \frac{1}{2}(\hat{c}_2^C - \hat{c}_2^D)(\hat{c}_2^C + \hat{c}_2^D) - \frac{1}{2}(\hat{c}_1^D - \hat{c}_1^C)(\hat{c}_1^C + \hat{c}_1^D) \le 0 \ (= 0 \text{ iff } v_1 = v_2).$$

Proof of Proposition 3.

Solver 1's ex ante expected payoff under full concealment minuses that under full disclosure is

$$\frac{\left(\hat{c}_{1}^{C}\right)^{2}}{2} - \frac{\left(\hat{c}_{1}^{D}\right)^{2}}{2} = \frac{\left(\hat{c}_{1}^{C} - \hat{c}_{1}^{D}\right)\left(\hat{c}_{1}^{C} + \hat{c}_{1}^{D}\right)}{2} < 0.$$

Solver 2's ex ante expected payoff under full concealment minuses that under full disclosure is

$$\frac{\left(\hat{c}_{2}^{C}\right)^{2}}{2} - \frac{\left(\hat{c}_{2}^{D}\right)^{2}}{2} = \frac{\left(\hat{c}_{2}^{C} - \hat{c}_{2}^{D}\right)\left(\hat{c}_{2}^{C} + \hat{c}_{2}^{D}\right)}{2} > 0.$$

Solver 1's probability of winning under full concealment minuses that under full disclosure is

$$\begin{aligned} \hat{c}_{1}^{C} \left[\hat{c}_{2}^{C} \cdot \frac{\hat{c}_{2}^{C} v_{1}}{2 \hat{c}_{1}^{C} v_{2}} + (1 - \hat{c}_{2}^{C}) \right] &- \hat{c}_{1}^{D} \left[\hat{c}_{2}^{D} \cdot \left(1 - \frac{v_{2}}{v_{1}} + \frac{v_{2}}{2v_{1}} \right) + (1 - \hat{c}_{2}^{D}) \right] \\ < \hat{c}_{1}^{D} \left[\hat{c}_{2}^{D} \cdot \frac{\hat{c}_{2}^{C} v_{1}}{2 \hat{c}_{1}^{C} v_{2}} + (1 - \hat{c}_{2}^{D}) \right] - \hat{c}_{1}^{D} \left[\hat{c}_{2}^{D} \cdot \left(1 - \frac{v_{2}}{v_{1}} + \frac{v_{2}}{2v_{1}} \right) + (1 - \hat{c}_{2}^{D}) \right] \\ = - \frac{\hat{c}_{1}^{D} v_{2} (1 - v_{1}) (1 + 2v_{1} - v_{2}) (v_{1} - v_{2})}{2v_{1} (1 - v_{2}) (1 + v_{2}) (1 - v_{1} + v_{2})} \\ < 0. \end{aligned}$$

Solver 2's probability of winning under full concealment minuses that under full disclosure

$$\begin{split} & \hat{c}_2^C \left[\hat{c}_1^C \cdot \left(1 - \frac{\hat{c}_2^C v_1}{\hat{c}_1^C v_2} + \frac{\hat{c}_2^C v_1}{2\hat{c}_1^C v_2} \right) + (1 - \hat{c}_1^C) \right] - \hat{c}_2^D \left[\hat{c}_1^D \cdot \frac{v_2}{2v_1} + (1 - \hat{c}_1^D) \right] \\ & > \hat{c}_2^D \left[\hat{c}_1^C \cdot \left(1 - \frac{\hat{c}_2^C v_1}{\hat{c}_1^C v_2} + \frac{\hat{c}_2^C v_1}{2\hat{c}_1^C v_2} \right) + (1 - \hat{c}_1^C) \right] - \hat{c}_2^D \left[\hat{c}_1^D \cdot \frac{v_2}{2v_1} + (1 - \hat{c}_1^D) \right] \\ & = \frac{\hat{c}_2^D (v_1 - v_2) (v_1 + 2v_1^2 + v_1 v_2 - v_2^2 - v_1 v_2^2)}{2v_1 (1 - v_2) (1 + v_1) (1 + v_2)} \\ > 0. \end{split}$$

is